A FIXED POINT THEOREM OF KRASNOSELSKII-SCHAEFER TYPE AND ITS APPLICATIONS IN CONTROL AND PERIODICITY OF INTEGRAL EQUATIONS

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Abstract. In this paper, we prove a fixed point theorem for the sum of a nonlinear contraction mapping and compact operator. The fixed point theorem obtained here resembles that of Krasnoselskii in which the mapping function is a combination of contraction and compact operators. It also takes the form of Schaefer's fixed point theorem of continuation type. Criteria on periodicity and control in integral equations are obtained by applying the fixed point theorem established.
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