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WEAK CONDITIONS FOR EXISTENCE OF RANDOM FIXED POINTS

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Abstract. In this article some results on existence of random fixed point for multivalued functions are extended. Such extensions involve weaker versions of some common concepts used on related studies.
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1. Introduction

Different crucial works have been published on random fixed points for multivalued random operators, such as the results presented by Beg and Shahzad [3], Engl [5], Itoh [9] and [10], Papageorgiou [15], Sehgal and Sing [16], Tan and Yuang [19] and [20], Xu [22] and Yuan and Yu [23]. Furthermore, Shahzad [18] and Kuman and Plubtieng [12], and Fierro et al. [8] recently obtained results on this subject. All the latter are closely related and some of them represent extensions of other works here included. This work contributes to this direction, as some of the previous results are here extended. The results of this study are based on new concepts, which were obtained from the weakening of some conditions and definitions usually used in the literature on Random Fixed Point. Thus, some conditions for the multivalued functions, based on their semicontinuity, are replaced by weaker conditions. As a matter of fact, Condition (A), which was introduced by Shahzad in [17], can be weakened and the same results will be obtained. For instance, Theorem 2.1, developed by this author, is a particular and direct consequence of our main result.

2. Preliminaries

In what follows, $\mathcal{F}$ will denote a σ-algebra of subsets of a set Ω. The symbol 2^E denotes the class of nonempty subsets of a set E.
For a mapping $F : X \to 2^Y$ and $B \subseteq Y$ let

$$F^{-1}(B) = \{x \in X : F(x) \cap B \neq \emptyset\}.$$

Let us suppose X and Y are topological spaces. The function F is said to be lower (upper) semicontinuous, if for each open (closed) subset B of Y, $F^{-1}(B)$ is an open (closed) subset of X. When F is lower and upper semicontinuous, then it is affirmed F is continuous. The function $\overline{F}$ from X to 2^Y, is defined by $\overline{F}(x) = \overline{F(x)}$, where $\overline{B}$ denotes the closure of a subset B of Y, and if Y is a topological vector space, $\overline{\text{co}}(F)$ stands for the function from X to 2^Y defined by $\overline{\text{co}}(F)(x) = \overline{\text{co}(F(x))}$, where $\text{co}(B)$ represents the convex hull of B. A multifunction $H : \Omega \to 2^X$ is said to be measurable, if for each open set A of X, $H^{-1}(A) \in \mathcal{F}$.

In the sequel, (X, d) denotes a metric space and whenever E is a subset of X, τ_E denotes the topology of E induced by d. In this work we use the definition of condensing function given by Tarafdar et al. in [21]. Indeed, $F : E \to 2^X$ is said to be condensing, if for each subset C of E such that $\gamma(C) > 0$, one has $\gamma(F(C)) < \gamma(C)$, where $F(C) = \bigcup_{x \in C} F(x)$ and γ is the Kuratowski measure of noncompactness, i.e., for each bounded subset A of E,

$$\gamma(A) = \inf\{\epsilon > 0 : A \text{ is covered by a finite number of sets of diameter } \leq \epsilon\}.$$

If A is not a bounded subset of E, we assign the measure of noncompactness of A to be infinity, i.e. $\gamma(A) = \infty$.

Let E be a subset of X, $F : E \to 2^X$ and $h_F : E \to \mathbb{R}$ be the mapping $h_F(x) = d(x, F(x))$. As in [17], F is said to be hemicompact, if and only if, every sequence $(x_n; n \in \mathbb{N})$ in E satisfying $h_F(x_n) \to 0$, has a convergent subsequence. We say F is weakly lower (upper) semicontinuous, if h_F is upper (lower) semicontinuous, and we say F is weakly continuous, if F is both weakly lower and weakly upper semicontinuous.

Let E be a subset of X and $\mathcal{C}$ a subfamily of 2^E. We say τ_E is σ-generated by $\mathcal{C}$, if for each $x \in E$, $\{x\} \in \mathcal{C}$ and for each nonempty open subset A of E, there exists a sequence $(C_n; n \in \mathbb{N})$ in $\mathcal{C}$ such that $A = \bigcup_{n=0}^{\infty} C_n$. Hence, a multifunction $H : \Omega \to 2^E$ is measurable, whenever for each $C \in \mathcal{C}$, $H^{-1}(C) \in \mathcal{F}$. Note that whenever E is separable, τ_E is σ-generated by the family of all closed balls of E, and whenever E is separable and locally compact, τ_E is σ-generated by the family of nonempty compact subsets of E.

Let E be a subset of X, $\mathcal{C}$ a subfamily of 2^E and $F : E \to 2^X$. We say F is $\mathcal{C}$-almost hemicompact, if τ_E is σ-generated by $\mathcal{C}$ and for each sequence $(x_n; n \in \mathbb{N})$ in E and $C \in \mathcal{C}$ such that $d(x_n, C) + h_F(x_n) \to 0$, there exists $x \in C$ such that $h_F(x) = 0$. Note that, for multivalued functions having nonempty and closed images, Condition (A) introduced by Shahzad in [17] is equivalent to be $\mathcal{C}$-almost hemicompact, where $\mathcal{C}$ is the family of all nonempty closed subsets of E.

It is easy to see that every F weakly upper semicontinuous and condensing is $\mathcal{C}$-almost hemicompact for each subfamily $\mathcal{C}$ of 2^E that σ-generates τ_E and such that for each $C \in \mathcal{C}$, C is closed. This fact is used in Theorem 3.9.

3. Main results

Proposition 3.1. *Let $E \subseteq X$ and $F : E \to 2^X$.*

(3.1.1) *If F is lower semicontinuous, then F is weakly lower semicontinuous.*
(3.1.2) *If F is upper semicontinuous, then F is weakly upper semicontinuous.*

Proof. Let suppose F is lower semicontinuous. Let $\alpha > 0$ and $A = \{x \in E : d(x, F(x)) < \alpha\}$. In order to prove that A is an open set, let suppose $A \neq \emptyset$ and choose $a \in A$. Let $\epsilon = \alpha - d(a, F(a))$ and $y \in F(a)$ such that $d(a, y) < \epsilon/3 + d(a, F(a))$. Since F is lower semicontinuous, there exists a neighborhood $U'(a)$ of a such that $F(u) \cap B(y, \epsilon/3) \neq \emptyset$ for all $u \in U'(a)$. Let $U(a) = U'(a) \cap B(a, \epsilon/3)$, $u \in U(a)$ and $b_u \in F(u) \cap B(y, \epsilon/3)$. One has $d(u, b_u) \leq d(u, a) + d(a, y) + d(y, b_u) < \alpha$ and consequently, $d(u, F(u)) < \alpha$. This proves that A is an open set and therefore, (3.1.1) holds.

Next suppose F is upper semicontinuous. Let $\alpha \in \mathbb{R}$ and $A = \{x \in E : d(x, F(x)) > \alpha\}$. Let us prove A is an open set. Let $a \in A$ and choose $\beta, \gamma \in \mathbb{R}$ such that $\gamma > \beta > \alpha$ and $d(a, F(a)) > \gamma$. Let $G = \{y \in E : d(y, F(a)) < (\gamma - \beta)/2\}$. Since $F(a) \subseteq G$, G is open and F is upper semicontinuous, there exists $U'(a)$ neighborhood of a such that for each $x \in U'(a)$, $F(x) \subseteq G$. This implies that for each $x \in U'(a)$ and each $y \in F(x)$, $d(y, F(a)) < (\gamma - \beta)/2$. Hence,

$$\gamma < d(a, F(a)) \leq d(a, y) + d(y, F(a)) < d(a, y) + (\gamma - \beta)/2.$$

Thus, $d(a, y) > (\gamma + \beta)/2$ and consequently, $d(a, F(x)) \geq (\gamma + \beta)/2$. Let $U(a) = U'(a) \cap B(a, \beta - \alpha)$ and note that for each $x \in U(a)$,

$$\beta < d(a, F(x)) \leq d(a, x) + d(x, F(x)) < \beta - \alpha + d(x, F(x)).$$

This proves that $U(a) \subseteq A$ and therefore, h_F is lower semicontinuous, which concludes the proof. $\square$

Remarks.

(R1) Let $F : \mathbb{R} \to 2^{\mathbb{R}}$ and $G : \mathbb{R} \to 2^{\mathbb{R}}$ the multivalued functions defined by

$$F(x) = \begin{cases} \{0\} & \text{si} \quad x \neq 0 \\ [-1, 1] & \text{si} \quad x = 0, \end{cases} \qquad \text{and} \qquad G(x) = \begin{cases} [-1, 1] & \text{si} \quad x \neq 0 \\ \{0\} & \text{si} \quad x = 0. \end{cases}$$

From page 39 in [1], F is not lower semicontinuous and G is not upper semicontinuous. However, h_F and h_G are continuous, indeed, for each $x \in \mathbb{R}$, $h_F(x) = |x|$ and

$$h_G(x) = \begin{cases} |x| - 1 & \text{if} \quad |x| > 1 \\ 0 & \text{if} \quad |x| \leq 1. \end{cases}$$

Hence, F and G are weakly continuous.

(R2) Let $E \subseteq X$ and $F : E \to 2^X$. Since $h_{\overline{F}} = h_F$, F is weakly lower or upper semicontinuous, if and only if, $\overline{F}$ is so.

(R3) If $F : E \subseteq X \to 2^X$ is hemicompact and weakly upper semicontinuous, then F is $\mathcal{C}$-almost hemicompact, for each $\mathcal{C} \subseteq CD(E)$ such that τ_E is σ-generated by $\mathcal{C}$, where $CD(E)$ is the family of all closed subsets of E.

Proposition 3.2. *Let $E \subseteq X$, $F : E \to 2^X$ weakly upper semicontinuous, $\mathcal{K}$ the family of nonempty compact subsets of E and suppose τ_E is σ-generated by $\mathcal{K}$. Then, F is $\mathcal{K}$-almost hemicompact.*

Proof. Let $(x_n; n \in \mathbb{N})$ be a sequence in E and let $K \in \mathcal{K}$ such that $d(x_n, K) + h_F(x_n) \to 0$. Since $d(x_n, K) \to 0$, there exist an increasing sequence $(n_k; k \in \mathbb{N})$ in $\mathbb{N}$ and $x \in K$ such that $x_{n_k} \to x$. Hence, $h_F(x) \leq \liminf h_F(x_{n_k}) = 0$ and therefore, $h_F(x) = 0$. □

Theorem 3.3. *Let E be a complete and separable subset of X, $F : \Omega \times E \to 2^X$ be a multifunction and $\mathcal{C} \subseteq 2^E$. Suppose the following two conditions hold:*

(3.3.1) *For each $\omega \in \Omega$, $F(\omega, \cdot)$ is weakly lower semicontinuous, $\mathcal{C}$-almost hemicompact and there exists $x_\omega \in E$ such that $x_\omega \in \overline{F}(\omega, x_\omega)$.*

(3.3.2) *For each $x \in E$, $F(\cdot, x)$ is measurable.*

Then, there exists $\xi : \Omega \to X$ measurable such that for each $\omega \in \Omega$, $\xi(\omega) \in \overline{F}(\omega, \xi(\omega))$.

Proof. Let $H : \Omega \to 2^E$ such that $H(\omega) = \{x \in E : x \in \overline{F}(\omega, x)\}$. Thus, $H(\omega) \neq \emptyset$. Since for each $x \in E$, $\{x\} \in \mathcal{C}$ and $F(\omega, \cdot)$ is $\mathcal{C}$-almost hemicompact, $H(\omega)$ is closed. For each $x \in E$, $F(\cdot, x)$ is measurable and hence the function $h_F : \Omega \times E \to \mathbb{R}$ defined by $h_F(\omega, x) = d(x, F(\omega, x))$ is measurable at the first variable. For each $C \in \mathcal{C}$ define $D_n = \{x \in E : d(x, C) < 1/n\} \cap D$, $(n \geq 1)$, where D is a dense and countable subset of E.

Let $L(C) = \bigcap_{n \geq 1} \bigcup_{x \in D_n} \{\omega \in \Omega : h_F(\omega, x) < 1/n\}$. Hence, $L(C)$ is measurable and by defining $B = \bigcap_{n \geq 1} \bigcup_{x \in C} \{\omega \in \Omega : h_F(\omega, x) < 1/n\}$ one has $H^{-1}(C) \subseteq B$. Let us prove that $B \subseteq L(C)$ to obtain $H^{-1}(C) \subseteq L(C)$. Let $\omega \in B$. For each $n \geq 1$ there exists $x_n \in C$ such that $h_F(\omega, x_n) < 1/n$. Let $G_n(\omega) = \{x \in X : h_F(\omega, x) < 1/n\} \cap B(x_n, 1/n)$. The upper semicontinuity of $h_F(\omega, \cdot)$ implies that $G_n(\omega)$ is a neighborhood of x_n and hence, there exists $x \in G_n(\omega) \cap D(C)$. That is $h_F(\omega, x) < 1/n$ and $x \in D_n$, which proves that $\omega \in L(C)$ and consequently $H^{-1}(C) \subseteq L(C)$.

Let us prove that $L(C) \subseteq H^{-1}(C)$. Let $\omega \in L(C)$. Thus, for each $n \geq 1$ there exists $x_n \in D_n$ such that $h_F(\omega, x_n) < 1/n$ and $d(x_n, C) < 1/n$. Since $F(\omega, \cdot)$ is $\mathcal{C}$-almost hemicompact, there exists $x \in C$ such that $h_F(x) = 0$. That is, $x \in H(\omega) \cap C$ and therefore, $\omega \in H^{-1}(C)$.

It has been proved that for each $C \in \mathcal{C}$, $H^{-1}(C) = L(C)$ and thus, H is measurable. It follows from the Kuratowsky and Ryll-Nardzewski theorem (cf. [13]) that H has a measurable selector ξ, which complete the proof. □

The above result is indeed a strict generalization of Theorem 2.1 by Shahzad in [17] since there are set valued functions which are not continuous and however they are weakly continuous. Furthermore, according to (R1), there exist non-semicontinuous (lower and upper) multifunctions which are weakly continuous. Even, the corollary below is also an extension of this theorem since its author considers the function F being continuous and the family $\mathcal{C}$ as the family of all closed subsets of E.

Corollary 3.4. *Let E be a complete and separable subset of X, $F : \Omega \times E \to 2^X$ be a multifunction and $\mathcal{C} \subseteq 2^E$. Suppose the following two conditions hold:*

(3.4.1) *For each $\omega \in \Omega$, $F(\omega, \cdot)$ is lower semicontinuous, $\mathcal{C}$-almost hemicompact and there exists $x_\omega \in E$ such that $x_\omega \in \overline{F}(\omega, x_\omega)$.*

(3.4.2) *For each $x \in E$, $F(\cdot, x)$ is measurable.*

Then, there exists $\xi : \Omega \to X$ measurable such that for each $\omega \in \Omega$, $\xi(\omega) \in \overline{F}(\omega, \xi(\omega))$.

Proof. It directly follows from the preceding theorem and Proposition 3.1. □

Corollary 3.5. *Let E be a locally compact, separable and complete subset of X and $F : \Omega \times E \to 2^X$ be a multifunction. Suppose the following two conditions hold:*

(3.5.1) *For each $\omega \in \Omega$, $F(\omega, \cdot)$ is weakly continuous and there exists $x_\omega \in E$ such that $x_\omega \in \overline{F}(\omega, x_\omega)$.*

(3.5.2) *For each $x \in E$, $F(\cdot, x)$ is measurable.*

Then, there exists $\xi : \Omega \to X$ measurable such that for each $\omega \in \Omega$, $\xi(\omega) \in \overline{F}(\omega, \xi(\omega))$.

Proof. Let $\mathcal{K}$ be the family of all nonempty compact subsets of E. It is clear that for each $x \in E$, $\{x\} \in \mathcal{K}$ and since E is locally compact and separable, it follows from Theorem 7.2 (page 241) in [6] that τ_E is σ-generated by $\mathcal{K}$. Moreover, for each $\omega \in \Omega$, $F(\omega, \cdot)$ is weakly upper semicontinuous and thus, by Proposition 3.2, $F(\omega, \cdot)$ is $\mathcal{K}$-almost hemicompact. Therefore, the conclusion of this corollary is obtained from Theorem 3.3. □

Recently, by following a Caristi's idea, Khamsi proved a theorem (Theorem 4 in [11]) which we use to obtain another result on random fixed point below.

Corollary 3.6. *Let E be a complete and separable metric space, $F : \Omega \times E \to 2^E$ be a multifunction, $\phi : E \to [0, \infty[$ a lower semicontinuous function and $\mathcal{C} \subseteq 2^E$. Suppose the following two conditions hold:*

(3.6.1) *For each $\omega \in \Omega$, $F(\omega, \cdot)$ is weakly lower semicontinuous, $\mathcal{C}$-almost hemicompact and for all $x \in E$ there exists $y \in \overline{F}(\omega, x)$ such that $d(x, y) \leq \phi(x) - \phi(y)$.*

(3.6.2) *For each $x \in E$, $F(\cdot, x)$ is measurable.*

Then, there exists $\xi : \Omega \to X$ measurable such that for each $\omega \in \Omega$, $\xi(\omega) \in \overline{F}(\omega, \xi(\omega))$.

Proof. It directly follows from Theorem 3.3 and Theorem 4 in [11]. □

The following result generalizes or improves known results in the literature.

Lemma 3.7. *Let (X, d) be a metric space, and $(x_n; n \in \mathbb{N})$ and $(y_n; n \in \mathbb{N})$ be two sequences in X such that $d(x_n, y_n) \to 0$. Then $\gamma(A) = \gamma(B)$, where $A = \{x_n : n \in \mathbb{N}\}$ and $B = \{y_n : n \in \mathbb{N}\}$.*

Proof. Let $\mathrm{B}(A, \epsilon) = \{y \in X : d(y, A) < \epsilon\}$. Since $d(x_n, y_n) \to 0$, given $\epsilon > 0$, there exists $N \in \mathbb{N}$, such that $d(x_n, y_n) < \epsilon$, for any $n \geq N$. Thus $d(y_n, A) \leq d(y_n, x_n) < \epsilon$ and consequently, for each $n \geq N$, $y_n \in \mathrm{B}(A, \epsilon)$. We have, $B \subseteq \{y_0, \ldots, y_N\} \cup \mathrm{B}(A, \epsilon)$ and hence $\gamma(B) \leq \gamma(\mathrm{B}(A, \epsilon)) \leq \gamma(A) + 2\epsilon$. This fact implies that $\gamma(B) \leq \gamma(A)$ and analogously $\gamma(A) \leq \gamma(B)$. Therefore, the proof is complete. □

Lemma 3.8. *Let (X, d) be a metric space, E a subset of X, $\mathcal{C}$ the family of all closed subsets of E such that τ_E is σ-generated by $\mathcal{C}$ and $F : E \to 2^X$ a weakly upper semicontinuous and condensing multivalued function. Then, F is $\mathcal{C}$-almost hemicompact.*

Proof. Let $C \in \mathcal{C}$ and $(x_n; n \in \mathbb{N})$ a be a sequence in E such that $d(x_n, C) + d(x_n, F(x_n)) \to 0$ as $n \to \infty$. Let $A = \{x_n : n \in \mathbb{N}\}$ and suppose $\gamma(A) > 0$. Since $d(x_n, F(x_n)) \to 0$ as $n \to \infty$, there exists a sequence $(y_n; n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $y_n \in F(x_n)$ and hence, $d(x_n, y_n) \to 0$. It follows from Lemma 3.7 that $\gamma(A) = \gamma(B)$, where $B = \{y_n : n \in \mathbb{N}\}$. Since F is condensing, $\gamma(\cup_{n \in \mathbb{N}} F(x_n)) < \gamma(A) = \gamma(B)$, which is a contradiction due to $B \subseteq \cup_{n \in \mathbb{N}} F(x_n)$. Consequently, $\gamma(A) = 0$ and thus, $(x_n; n \in \mathbb{N})$ has a subsequence $(x_{n_k}; k \in \mathbb{N})$ converging to a point $x_0 \in X$. But $d(x_0, C) \leq d(x_0, x_{n_k}) + d(x_{n_k}, C)$ and hence $d(x_0, C) = 0$. Since C is closed, $x_0 \in C$. Moreover, F is weakly upper semicontinuous and consequently

$$d(x_0, F(x_0)) \leq \liminf_{k \to \infty} d(x_{n_k}, F(x_{n_k})) = 0.$$

Therefore, F is $\mathcal{C}$-almost hemicompact, which concludes the proof. □

The following result generalizes or improves known results in the literature.

Theorem 3.9. *Let E be a nonempty, bounded, closed, convex and separable subset of a Banach space X and $F : \Omega \times E \to 2^E$ be a multifunction with convex images satisfying the following two conditions:*

(3.9.1) *For each $\omega \in \Omega$, $F(\omega, \cdot)$ is lower semicontinuous, weakly upper semicontinuous and condensing.*

(3.9.2) *For each $x \in E$, $F(\cdot, x)$ is measurable.*

Then, there exists $\xi : \Omega \to E$ measurable such that for each $\omega \in \Omega$, $\xi(\omega) \in \overline{F}(\omega, \xi(\omega))$.

Proof. Let $\mathcal{C}$ be the family of all closed subsets of E and fix $\omega \in \Omega$. Since $F(\omega, \cdot)$ is condensing and weakly upper semicontinuous, from Lemma 3.8 it is $\mathcal{C}$-almost hemicompact. From Corollary 3.4, it remains to prove that there exist $x_\omega \in E$ such that $x_\omega \in \overline{F}(\omega, x_\omega)$.

Lemma 3 by Tarafdar et al. in [21] implies that there exists a nonempty compact convex subset $K(\omega)$ of E such that for each $x \in K$, $F(\omega, x) \subset K(\omega)$. We continue denoting by $F(\omega,\cdot)$ the restriction of $F(\omega,\cdot)$ to $K(\omega)$. On the other hand, from (3.9.1), (3.9.2), and Lemma 2 in [7], $\overline{F}(\omega,\cdot) : K(\omega) \to 2^E$ is lower semicontinuous and hence by the Michael Selection Theorem [14], there exists $f_\omega : K(\omega) \to E$ continuous such that for each $x \in K(\omega)$, $f_\omega(x) \in \overline{F}(\omega, x)$. We have $f_\omega(K(\omega)) \subset K(\omega)$ and consequently the Schauder Theorem (see for instance, Theorem 9.5, Chapter V in [4]) implies that there exists $x_\omega \in K(\omega)$ such that $x_\omega = f_\omega(x_\omega) \in \overline{F}(\omega, x_\omega)$, which concludes the proof. □

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