

## A NONLINEAR INTEGRAL EQUATION VIA PICARD OPERATORS

CECILIA CRĂCIUN\* AND MARCEL-ADRIAN ȘERBAN\*

\* Department of Applied Mathematics  
Babeș-Bolyai University of Cluj-Napoca  
M. Kogălniceanu Street No.1, 400048-Cluj-Napoca  
Romania

E-mails: ceciliacraciun@yahoo.com mserban@math.ubbcluj.ro

**Abstract.** In this paper we study the following mixed type Volterra-Fredholm functional integral equation

$$x(t) = F\left(t, x(t), \int_{a_1}^{t_1} \dots \int_{a_m}^{t_m} K(t, s, x(s)) ds, \int_{a_1}^{b_1} \dots \int_{a_m}^{b_m} H(t, s, x(s)) ds\right).$$

Using the Picard operator technique we establish existence, uniqueness, data dependence and Gronwall results for the solutions. Also, it is studied the Ulam-Hyers stability of this equation.

**Key Words and Phrases:** Picard operators, mixed type Volterra-Fredholm functional integral equation, data dependence, comparison theorem, Ulam-Hyers stability, Gronwall lemma, operatorial inequalities.

**2010 Mathematics Subject Classification:** 47H10, 45G10, 35L10.

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*Received: October 27, 2009; Accepted: January 13, 2010.*