

## CONVERGENCE CRITERIA OF GENERALIZED HYBRID PROXIMAL POINT ALGORITHMS

LU-CHUAN CENG<sup>\*,1</sup> AND JUEI-LING HO<sup>\*\*</sup>

<sup>\*</sup>Department of Mathematics, Shanghai Normal University  
Shanghai 200234, and  
Scientific Computing Key Laboratory of Shanghai Universities, China  
E-mail: zenglc@hotmail.com

<sup>\*\*</sup>Department of Finance  
Tainan University of Technology, Tainan, Taiwan  
E-mail: t20054@mail.tut.edu.tw

**Abstract.** In this paper, we introduce and analyze some generalized hybrid proximal point algorithms for finding a common element of the set of zeros of a maximal monotone operator and the set of fixed points of a nonexpansive mapping in a Hilbert space. These algorithms include the previously known proximal point algorithms as special cases. Weak and strong convergence of the proposed proximal point algorithms are proved under some mild conditions.

**Key Words and Phrases:** Maximal monotone operator, nonexpansive mapping, zero point, fixed point, proximal point algorithm, resolvent identity, demiclosedness principle, Opial's property, convergence analysis.

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