EXISTENCE OF POSITIVE SOLUTIONS OF NEUMANN BOUNDARY VALUE PROBLEM VIA A CONVEX FUNCTIONAL COMPRESSION-EXPANSION FIXED POINT THEOREM

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Abstract. This paper is devoted to study the existence of positive solutions of second-order boundary value problem

$$-u'' + Mu = h(t)f(t, u), \quad t \in (0, 1)$$

with Neumann boundary conditions

$$u'(0) = u'(1) = 0,$$

where $M > 0$, $f \in C([0,1] \times \mathbb{R}^+, \mathbb{R}^+)$. $h(t)$ is allowed to be singular at $t = 0$ and $t = 1$. The arguments are based only upon the positivity of the Green's function and the fixed point theorem of cone expansion and compression of convex function type.

Key Words and Phrases: Neumann BVP, positive solutions, cone, fixed point theorem.

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