Fixed Point Theory, 11(2010), No. 2, 383-394
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

STRUCTURE OF COMMON FIXED POINT SET OF DEMICONTINUOUS ASYMPTOTICALLY S-NONEXPANSIVE SEMIGROUPS

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Dedicated to Wataru Takahashi on the occasion of his retirement

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Abstract. Every asymptotically nonexpansive mapping is uniformly continuous, but this fact is not true for asymptotically S-nonexpansive mappings in general. In a Banach space X, by constructing a sequence $\{x_n\}$ defined in Browder's technique for a demicontinuous asymptotically S-nonexpansive semigroup $\mathcal{T} = \{T(t) : t \geq 0\}$ of mappings from $C \subset X$ into itself with function $k(\cdot)$, we prove that the common fixed point set $F(S) \cap \cap_{t>0} F(T(t))$ is the sunny nonexpansive retract of $F(S)$, where $F(S)$ is the fixed point set of a weakly continuous mapping S form C into itself. Under the assumption on uniform convexity of X, we prove that the common fixed point set $F(S) \cap \cap_{t>0} F(T(t))$ is closed and convex.

Key Words and Phrases: Asymptotically S-nonexpansive mapping, commutativity, S-contraction mapping, S-Lipschitzian semigroup, weakly continuous mapping, weakly continuous duality mapping.

2010 Mathematics Subject Classification: 47H09, 46B20, 47H10, 54H25.

References

[1] R.P. Agarwal, D. O'Regan, D.R. Sahu, *Fixed Point Theory for Lipschitzian-type Mappings with Applications*, Series: Topological Fixed Point Theory and Its Applications, **6**, Springer, New York, 2009.

[2] F.E. Browder, *Convergence of approximants to fixed points of nonexpansive nonlinear mappings in Banach spaces*, Arch. Rational Mech. Anal., **24**(1967), 82-90.

[3] F.E. Browder, *Convergence theorems for sequences of nonlinear operators in Banach spaces*, Math. Z., **100**(1967), 201-225.

[4] M A. Al-Thagafi, *Common fixed points and best approximation*, J. Approx. Theory, **85**(1996), 318-323.

[5] I. Beg, D.R. Sahu, S.D. Diwan, *Approximation of fixed points of uniformly R-subweakly commuting mappings*, J. Math. Anal. Appl., **324**(2006), No. 2, 1105-1114.

[6] K. Goebel, W A. Kirk, *A fixed point theorem for asymptotically nonemxpansive mappings*, Proc. Amer. Math. Soc., **35**(1972), 171-174.

The paper was presented at The 9th International Conference on Fixed Point Theory and Its Applications, July 16-22, 2009, National Changhua University of Education, Changhua, Taiwan (R.O.C.).

[7] W.J. Dotson Jr., *Fixed point theorems for nonexpansive mappings on starshaped subsets of Banach spaces*, J. London Math. Soc., **4**(1972), 408-410.
[8] G. Jungck, *Commuting mappings and fixed points*, Amer. Math. Monthly, **83**(1976), 261-263.
[9] G. Jungck, *Coincidence and fixed points for compatible and relatively nonexpansive maps*, Internat. J. Math. and Math. Sci., **16**(1993), No. 1, 95-100.
[10] C.H. Morales, *Strong convergence of path for continuous pseudo-contractive mappings*, Proc. Amer. Math. Soc., **135** (2007), 2831-2838.
[11] C.H. Morales, J.S. Jung, *Convergence of paths for pseudo-contractive mappings in Banach spaces*, Proc. Amer. Math. Soc., **128**(2000), 3411-3419.
[12] Z. Opial, *Weak convergence of successive approximations for nonexpansive mappings*, Bull. Amer. Math. Soc., **73**(1967), 591-597.
[13] S. Park, *On f-nonexpansive maps*, J. Korean Math. Soc., **16**(1979), 29-38.
[14] D.R. Sahu, Z. Liu, S.M. Kang, *Iterative approaches to common fixed points of asymptotically nonexpansive mappings*, Rocky Mountain J. Math., **39**(2009), 281-304.
[15] D.R. Sahu, D. O'Regan, *Convergence theorems for semigroup-type families of non-self mappings*, Rendi. del Circolo Mat. di Palermo, **57**(2008), 305-329.
[16] J. Schu, *Approximation of fixed points of asymptotically nonexpansive mappings*, Proc Amer. Math. Soc., **112**(1991), No. 1, 143-151.
[17] N. Shahzad, *Invariant approximations and R-subweakly commuting maps*, J. Math. Anal. Appl., **257**(2001), 39-45.
[18] S.P. Singh, B. Watson, P. Srivastava, *Fixed Point Theory and Best Approximation: The KKM-map Principle*, Series: Mathematics and Its Applications, Springer-Verlag, 1997.
[19] P. Vijayaraju, *Fixed point theorems for asymptotically nonexpansive mapping*, Bull. Cal. Math. Soc., **80**(1998), 133-136.
[20] N.C. Wong, D.R. Sahu, J.C. Yao, *Solving variational inequalities involving nonexpansive type mappings*, Nonlinear Anal., **69**(2008), 4732-4753.

Received: December 31, 2009; Accepted: May 2, 2010.