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MONOTONE GENERALIZED WEAK CONTRACTIONS IN PARTIALLY ORDERED METRIC SPACES

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Abstract. In this paper, a concept of monotone generalized contraction in partially ordered metric spaces is introduced and some fixed point and common fixed point theorems for the so-called weak contractions are proved. The concept of weak contraction was introduced by Kada, Suzuki and Takahashi [Math. Japonica, 44 (1996), 381-391], in connection to the concept of w-distance on a metric space. The results of the present paper represent extensions and improvements of some theorems given in the setting of partially ordered metric spaces by Nieto and Rodriguez-Lopez [Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, Order 22 (2005), 223-239; Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations, Acta Math. Sinica, 23 (2007) 2205-2212] and Ran and Reurings [A fixed point theorem in partially ordered sets and some applications to matrix equations, Proc. Amer. Math. Soc. 132 (2004), 1435-1443], to more general classes of contractive type mappings in partially ordered metric spaces.
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1. Introduction

The well-known Banach fixed point theorem for contraction mappings has been generalized and extended in many directions ([1]-[7], [11], [15], [16], [8], [17]-[18], [21], [25], [26], [19]). Recently Nieto and Rodriguez-Lopez [19], [20], Ran and Reurings [24] and Petruşel and Rus [22] presented some new results for contractions in partially ordered metric spaces. The multivalued case in the setting of an ordered complete gauge space was very recently discussed by G. Petruşel in [23]. The main idea in [19], [20], [24] is to combine the iterative procedures in the contraction principle with the monotone iterations technique.

Recall that if $(X, \leq)$ is a partially ordered set, then $F : X \to X$ is said to be non-decreasing if $x, y \in X, x \leq y$ implies $F(x) \leq F(y)$.

The main result of Nieto and Rodriguez-Lopez [19], [20] and Ran and Reurings [24] is the following fixed point theorem.

Theorem 1.1. *Let $(X, \leq)$ be a partially ordered set and suppose there is a metric d on X such that (X, d) is a complete metric space. Suppose $F : X \to X$ is a non-decreasing mapping and there exists $0 \leq k < 1$ such that $d(F(x), F(y)) \leq kd(x, y)$, for all $x, y \in X$ with $x \leq y$.*
Suppose that one of the following assertions holds:
(a) *F is continuous or*
(b) *if $\{x_n\} \subset X$ is a non-decreasing sequence with $x_n \to x$ in X, then $x_n \leq x$ for all $n \in \mathbb{N}$.*
If there exists an $x_0 \in X$ with $x_0 \leq F(x_0)$, then F has a fixed point.

In this paper, a concept of monotone generalized contraction in partially ordered metric spaces is introduced and some fixed point and common fixed point theorems for the so-called weak contractions are proved.

2. Preliminaries

Kada, Suzuki and Takahashi [13] introduced in 1996, the concept of w-distance on a metric space and proved some fixed point theorems. In the sequel, we state the definition of a w-distance and we state a lemma which we will use in the main sections of this work. For other details, we refer the reader to [13], [27] and [28].

Definition 2.1. ([13]) Let (X, d) be a metric space. Then a function $p : X \times X \longrightarrow [0, \infty)$ is called a w-distance on X if the following are satisfied:
(a) $p(x, z) \leq p(x, y) + p(y, z)$ for any $x, y, z \in X$;
(b) for any $x \in X$, $p(x, .) : X \longrightarrow [0, \infty)$ is lower semi-continuous;
(c) for any $\varepsilon > 0$, there exists $\delta > 0$ such that $p(z, x) \leq \delta$ and $p(z, y) \leq \delta$ imply $d(x, y) \leq \varepsilon$.

Let us recall that a real-valued function f defined on a metric space X is said to be lower semi-continuous at a point x_0 in X if either $\liminf_{x_n \to x_0} f(x_n) = \infty$ or $f(x_0) \leq \liminf_{x_n \to x_0} f(x_n)$, whenever $x_n \in X$ for each $n \in \mathbb{N}$ and $x_n \to x_0$ [12]).

Let us give some examples of w-distance.

Example 2.2. ([13]) Let (X, d) be a metric space. Then the metric d is a w-distance on X.

Example 2.3. ([13]) Let $(X, \|.\|)$ be a normed space. Then the function $p : X \times X \longrightarrow [0, \infty)$ defined by $p(x, y) = \|x\| + \|y\|$ for every $x, y \in X$ is a w-distance on X.

Example 2.4. ([13]) Let $(X, \|.\|)$ be a normed space. Then the function $p : X \times X \longrightarrow [0, \infty)$ defined by $p(x, y) = \|y\|$ for every $x, y \in X$ is a w-distance on X.

Example 2.5. ([30]) Let $X = \{a, b\}$. Then the function $p : X \times X \longrightarrow [0, \infty)$ defined by

$$p(x, y) = \begin{cases} 0, & \text{if } x = a \text{ and } y = b, \\ 1, & \text{otherwise,} \end{cases}$$

is a w-distance on X.

Lemma 2.6. ([13, 28]) *Let X be a metric space with metric d and p be a w-distance on X. Let $\{x_n\}$ and $\{y_n\}$ be sequences in X, let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[0,\infty)$ converging to zero, and let $x, y, z \in X$. Then the following hold:*

(1) *If $p(x_n, y) \leq \alpha_n$ and $p(x_n, z) \leq \beta_n$ for any $n \in \mathbb{N}$, then $y = z$. In particular, if $p(x, y) = 0$ and $p(x, z) = 0$, then $y = z$;*

(2) *if $p(x_n, y_n) \leq \alpha_n$ and $p(x_n, z) \leq \beta_n$ for any $n \in \mathbb{N}$, then $d(y_n, z) \to 0$;*

(3)*if $p(x_n, x_m) \leq \alpha_n$ for any $n, m \in \mathbb{N}$ with $m > n$, then $\{x_n\}$ is a Cauchy sequence;*

(4) *if $p(y, x_n) \leq \alpha_n$ for any $n \in \mathbb{N}$, then $\{x_n\}$ is a Cauchy sequence.*

3. Fixed point theorems

We introduce first the following concept.

Definition 3.1. Suppose $(X, \leq)$ is a partially ordered set and $f : X \to X$ be a self mapping on X. We say f is inverse increasing if for $x, y \in X$,

$$f(x) \leq f(y) \text{ implies } x \leq y. \tag{3.1}$$

Our first main result is a fixed point theorem for graphic contractions on a partially oredered metric space endowed with a w-distance.

Theorem 3.2. *Let $(X, \leq)$ be a partially ordered set and let $d : X \times X \to \mathbb{R}_+$ be a metric on X such that (X, d) is a complete metric space. Suppose that p is a w-distance in (X, d). Let $A : X \to X$ be a non-decreasing mapping and there exists $k \in [0, 1)$ such that*

$$p(Ax, A^2x) \leq kp(x, Ax), \quad \text{for all } x \leq Ax. \tag{3.2}$$

Suppose also that:

(i) for every $x \in X$ with $x \leq Ax$

$$\inf\{p(x, y) + p(x, Ax)\} > 0, \text{ for every } y \in X \text{ with } y \neq Ay. \tag{3.3}$$

(ii) there exists $x_0 \in X$ such that $x_0 \leq Ax_0$.

Then A has a fixed point in X.

Proof. If $Ax_0 = x_0$, then the proof is finished. Suppose that $Ax_0 \neq x_0$. Since $x_0 \leq Ax_0$ and A is non-decreasing, we obtain $x_0 \leq Ax_0 \leq A^2x_0 \leq ... \leq A^{n+1}x_0 \leq ....$ Hence, for each $n \in \mathbb{N}$ we have

$$p(A^n x_0, A^{n+1}x_0) \leq k^n p(x_0, Ax_0). \tag{3.4}$$

Then, for $n \in \mathbb{N}$ with $m > n$ we successively have

$$\begin{aligned} p(A^n x_0, A^m x_0) &\leq p(A^n x_0, A^{n+1}x_0) + \cdots + p(A^{m-1}x_0, A^m x_0) \\ &\leq k^n p(x_0, Ax_0) + \cdots + k^{m-1} p(x_0, Ax_0) \\ &\leq \frac{k^n}{1-k} p(x_0, Ax_0). \end{aligned}$$

By Lemma 2.6 (3), we conclude that $\{A^n x_0\}$ is Cauchy sequence in (X, d). Since (X, d) is a complete metric space, there exists $z \in X$ such that $\lim_{n\to\infty} A^n x_0 = z$.

Let $n \in \mathbb{N}$ be arbitrary but fixed. Then since $\{A^m x_0\}$ converges to z in (X, d) and $p(A^n x_0, \cdot)$ is lower semi-continuous, we have

$$p(A^n x_0, z) \leq \liminf_{m\to\infty} p(A^n x_0, A^m x_0) \leq \frac{k^n}{1-k} p(x_0, Ax_0).$$

Assume that $z \neq Az$. Since $A^n x_0 \leq A^{n+1} x_0$, by (3.3), we have

$$\begin{aligned} 0 &< \inf\{p(A^n x_0, z) + p(A^n x_0, A^{n+1} x_0)\} \\ &\leq \inf\{\frac{k^n}{1-k} p(x_0, Ax_0) + k^n p(x_0, Ax_0)\} = 0. \end{aligned}$$

This is a contradiction. Therefore, we have $z = Az$. □

Another result of this type is the following.

Theorem 3.3. *Let $(X, \leq)$ be a partially ordered set, let $d : X \times X \to \mathbb{R}_+$ be a metric on X such that (X, d) is a complete metric space. Suppose that p is a w-distance in (X, d). Let $A : X \to X$ be a non-decreasing mapping and there exists $k \in [0, 1)$ such that*

$$p(Ax, A^2 x) \leq k p(x, Ax), \quad \text{for all } x \leq Ax. \tag{3.5}$$

Assume that one of the following assertions holds:

(i) *for every $x \in X$ with $x \leq Ax$*

$$\inf\{p(x, y) + p(x, Ax)\} > 0, \text{ for every } y \in X \text{ with } y \neq Ay. \tag{3.6}$$

(ii) *if both $\{x_n\}$ and $\{Ax_n\}$ converge to y, then $y = Ay$;*

(iii) *A is continuous.*

If there exists $x_0 \in X$ with $x_0 \leq Ax_0$, then A has a fixed point in X.

Proof. The case (i), was proved in Theorem 3.2.
Let us prove first that (ii)$\Longrightarrow$(i). Assume that there exists $y \in X$ with $y \neq Ay$ such that $\inf\{p(x, y) + p(x, Ax) \ : \ x \leq Ax\} = 0$. Then there exists $\{z_n\} \in X$ such that $z_n \leq Az_n$ and $\lim_{n\to\infty}\{p(z_n, y) + p(z_n, Az_n)\} = 0$. Then $p(z_n, y) \longrightarrow 0$ and $p(z_n, Az_n) \longrightarrow 0$. By Lemma 2.6, we have that $Az_n \longrightarrow y$. We also have

$$\begin{aligned} p(z_n, A^2 z_n) &\leq p(z_n, Az_n) + p(Az_n, A^2 z_n) \\ &\leq (1 + k) p(z_n, Az_n) \longrightarrow 0 \text{ as } n \to \infty. \end{aligned}$$

Again by Lemma 2.6, we get $A^2 z_n \longrightarrow y$. Put $x_n = Az_n$. Then both $\{x_n\}$ and $\{Ax_n\}$ converges to y. Thus, by (ii) we have $y = Ay$. Thus (ii)$\Longrightarrow$(i) holds.
Now, we show that (iii)$\Longrightarrow$(ii). Let A be continuous. Further assume that $\{x_n\}$ and $\{Ax_n\}$ converges to y. Then we have $Ay = A(\lim_{n\to\infty} x_n) = \lim_{n\to\infty} x_n = y$. □

4. Common fixed point theorem for commuting mappings

The following theorem was given by Jungck [9] and it represents a generalization of the Banach contraction principle in complete metric spaces.

Theorem 4.1. *Let f be a continuous self mapping on a complete metric space (X, d) and let $g : X \longrightarrow X$ be another mapping, such that the following conditions are satisfied:*

(a) *$g(X) \subseteq f(X)$;*
(b) *g commutes with f;*
(c) *$d(g(x), g(y)) \leq kd(f(x), f(y))$, for all $x, y \in X$ and for some $0 \leq k < 1$.*

Then f and g have a unique common fixed point.

The next example shows that if the mapping $f : X \to X$ is continuous with respect to a metric d on X and $g : X \to X$ satisfies the condition

$$p(g(x), g(y)) \leq kp(f(x), f(y)), \text{ for all } x, y \in X \text{ and some } k \in [0, 1),$$

then, in general, g may be not continuous in (X, d).

Example 4.2. Let $X := (\mathbb{R}, |.|)$ be a normed linear space. Consider Example 2.4 with w-distance defined by

$$p(x, y) = |y| \text{ for every } x, y \in \mathbb{R}.$$

Consider the functions f and g defined by $f(x) = 4$ and

$$g(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Q}, \\ 0, & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Then $p(g(x), g(y)) = |g(y)| \leq 1 \leq \frac{1}{3}p(f(x), f(y)) = \frac{|f(y)|}{3} = \frac{4}{3}$.

Definition 4.3. Let $(X, \leq)$ be a partially ordered set and $g, h : X \to X$. By definition, we say that g is h-non-decreasing if for $x, y \in X$,

$$h(x) \leq h(y) \text{ implies } g(x) \leq g(y). \tag{4.1}$$

Our next result is a generalization of the above mentioned result of Jungck [9], for the case of a weak contraction with respect to a w-distance.

Theorem 4.4. *Let $(X, \leq)$ be a partially ordered set and let $d : X \times X \to \mathbb{R}_+$ be a metric on X such that (X, d) is a complete metric space. Suppose that p is a w-distance on X. Let $f, g : X \longrightarrow X$ be mappings that satisfy the following conditions:*

(a) *$g(X) \subseteq f(X)$;*
(b) *g is f-non-decreasing and f is inverse increasing;*
(c) *g commutes with f and f, g are continuous in (X, d);*
(d) *$p(g(x), g(y)) \leq kp(f(x), f(y))$ for all $x, y \in X$ with $x \leq y$ and some $0 < k < 1$.*
(e) *there exists $x_0 \in X$ such that (i) $f(x_0) \leq g(x_0)$ and (ii) $f(x_0) \leq f(g(x_0))$.*

Then f and g have a common fixed point $u \in X$. Moreover, if $g(v) = g^2(v)$ for all $v \in X$, then $p(u, u) = 0$.

Proof We claim that for every $f(x) \leq g(x)$

$$\inf\{p(f(x), g(x)) + p(f(x), z) + p(g(x), z) + p(g(x), g(g(x)))\} > 0$$

for every $z \in X$ with $g(z) \neq g(g(z))$. For the moment suppose the claim is true. Let $x_0 \in X$ with $f(x_0) \leq g(x_0)$. By (a) we can find $x_1 \in X$ such that $f(x_1) = g(x_0)$. By induction, we can define a sequence $\{x_n\}_n \in X$ such that

$$f(x_n) = g(x_{n-1}). \tag{4.2}$$

Since $f(x_0) \leq g(x_0)$ and $f(x_1) = g(x_0)$, we have

$$f(x_0) \leq f(x_1). \tag{4.3}$$

Then from (b), we get $g(x_0) \leq g(x_1)$, which means, by (4.2), that $f(x_1) \leq f(x_2)$. Again by (b) we get $g(x_1) \leq g(x_2)$, that is, $f(x_2) \leq f(x_3)$. By this procedure, we obtain

$$g(x_0) \leq g(x_1) \leq g(x_2) \leq g(x_3) \leq \cdots \leq g(x_n) \leq g(x_{n+1}) \leq \cdots. \tag{4.4}$$

Hence from (4.2) and (4.4) we have $f(x_{n-1}) \leq f(x_n)$ and by (3.1) we have $x_{n-1} \leq x_n$. By induction we get, for $n \geq 1$, that

$$\begin{aligned} p(f(x_n), f(x_{n+1})) &= p(g(x_{n-1}), g(x_n)) \\ &\leq kp(f(x_{n-1}), f(x_n)) \leq \cdots \leq k^n p(f(x_0), f(x_1)). \end{aligned}$$

This implies that, for $m, n \in \mathbb{N}$ with $m > n$,

$$\begin{aligned} & p(f(x_n), f(x_m)) \\ \leq\ & p(f(x_{m-1}, f(x_m)) + p(f(x_{m-2}), f(x_{m-1})) + \cdots + p(f(x_n), f(x_{n+1})) \\ \leq\ & p(f(x_0), f(x_1)) \sum_{j=n}^{m-1} k^j \leq \frac{k^n}{1-k} p(f(x_0), f(x_1)). \end{aligned}$$

Thus, by Lemma 2.6, we obtain that $\{f(x_n)\}$ is a Cauchy sequence in (X, d). Since (X, d) is complete, there exists $y \in X$ such that $\lim_{n\to\infty} f(x_n) = y$. As a result the sequence $g(x_{n-1}) = f(x_n)$ tends to y as $n \to +\infty$ and hence $\{g(f(x_n))\}_n$ converges to $g(y)$ as $n \to +\infty$. However, $g(f(x_n)) = f(g(x_n))$, by the commutativity of f and g, implies that $f(g(x_n))$ converges to $f(y)$ as $n \to +\infty$. Since the limit is unique, we get $f(y) = g(y)$ and, thus, $f(f(y)) = f(g(y))$. On the other hand, by lower semi-continuity of $p(x, \cdot)$ we have, for each $n \in \mathbb{N}$, that

$$p(f(x_n), y) \leq \liminf_{m\to\infty} p(f(x_n), f(x_m)) \leq \frac{k^n}{1-k} p(f(x_0), f(x_1)),$$

$$p(g(x_n), y) \leq \liminf_{m\to\infty} p(f(x_{n+1}), f(x_m)) \leq \frac{k^{n+1}}{1-k} p(f(x_0), f(x_1)).$$

Notice that, by (4.3), (4.2) and (4.1) we obtain $f(x_0) \leq f(f(x_1))$ and thus, by (4.1), we get $g(x_0) \leq g(f(x_1))$. Then $f(x_1) \leq g(f(x_1)) = f(g(x_1)) = f(f(x_2))$. By (4.1) we get that $g(x_1) \leq g(f(x_2))$ and thus $f(x_2) \leq f(g(x_2))$.
Continuing this process we get thet $f(x_n) \leq f(g(x_n))$, for $n \geq 0$, and, by (3.1), we get $x_n \leq g(x_n)$, for $n \geq 0$. Using now the condition (d), we have

$$\begin{aligned} p(g(x_n), g(g(x_n))) &\leq kp(f(x_n), f(g(x_n)) \\ &= kp(g(x_{n-1}), g(g(x_{n-1}))) \\ &\leq k^2 p(f(x_{n-1}), f(g(x_{n-1}))) \\ &= k^2 p(g(x_{n-2}), g(g(x_{n-2}))) \\ &\leq \cdots \leq k^n p(f(x_1), g(f(x_1))). \end{aligned}$$

We will show that $g(y) = g(g(y))$. Suppose, by contradiction, that $g(y) \neq g(g(y)))$. Then, we have:

$$\begin{aligned}
0 &< \inf\{p(f(x), g(x)) + p(f(x), y) + p(g(x), y) + p(g(x), g(g(x))) : x \in X\} \\
&\leq \inf\{p(f(x_n), g(x_n)) + p(f(x_n), y) + p(g(x_n), y) + p(g(x_n), g(g(x_n))) : n \in \mathbb{N}\} \\
&= \inf\{p(f(x_n), f(x_{n+1})) + p(f(x_n), y) + p(g(x_n), y) + p(g(x_n), g(g(x_n))) : n \in \mathbb{N}\} \\
&\leq \inf_n \{k^n p(f(x_0), f(x_1)) + \frac{k^n}{1-k} p(f(x_0), f(x_1)) + \frac{k^{n+1}}{1-k} p(f(x_0), f(x_1)) \\
&+ k^n p(f(x_1), g(f(x_1))) : n \in \mathbb{N}\} = 0.
\end{aligned}$$

This is a contradiction. Therefore $g(y) = g(g(y))$. Thus, $g(y) = g(g(y)) = f(g(y))$. Hence $u := g(y)$ is a common fixed point of f and g.

Furthermore, if $g(v) = g(g(v))$ for all $v \in X$, we have

$$\begin{aligned}
p(g(y), g(y)) &= p(g(g(y)), g(g(y))) \\
&\leq kp(f(g(y)), f(g(y))) = kp(g(y), g(y)),
\end{aligned}$$

which implies that, $p(g(y), g(y)) = 0$.

Now it remains to prove the initial claim. Assume that there exists $y \in X$ with $g(y) \neq g(g(y))$ and

$$\inf\{p(f(x), g(x)) + p(f(x), y) + p(g(x), y) + p(g(x), g(g(x))) : x \in X\} = 0.$$

Then there exists $\{x_n\}$ such that

$$\lim_{n\to\infty} \{p(f(x_n), g(x_n)) + p(f(x_n), y) + p(g(x_n), y) + p(g(x_n), g(g(x_n)))\} = 0.$$

Since $p(f(x_n), g(x_n)) \longrightarrow 0$ and $p(f(x_n), y) \longrightarrow 0$, by Lemma 2.6, we have

$$\lim_{n\to\infty} g(x_n) = y. \tag{4.5}$$

Also, since $p(g(x_n), y) \longrightarrow 0$ and $p(g(x_n), g(g(x_n))) \longrightarrow 0$, by Lemma 2.6, we have

$$\lim_{n\to\infty} g(g(x_n)) = y. \tag{4.6}$$

By (4.5), (4.6) and the continuity of g we have $g(y) = g(\lim_n g(x_n)) = \lim_n g(g(x_n)) = y$. Therefore, $g(y) = g(g(y))$, which is a contradiction. Hence, if $g(y) \neq g(g(y))$, then

$$\inf\{p(f(x), g(x)) + p(f(x), y) + p(g(x), y) + p(g(x), g(g(x))) : x \in X\} > 0. \quad \square$$

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