MONOTONE GENERALIZED WEAK CONTRACTIONS IN PARTIALLY ORDERED METRIC SPACES

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Abstract. In this paper, a concept of monotone generalized contraction in partially ordered metric spaces is introduced and some fixed point and common fixed point theorems for the so-called weak contractions are proved. The concept of weak contraction was introduced by Kada, Suzuki and Takahashi [Math. Japonica, 44 (1996), 381-391], in connection to the concept of w-distance on a metric space. The results of the present paper represent extensions and improvements of some theorems given in the setting of partially ordered metric spaces by Nieto and Rodriguez-Lopez [Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, Order 22 (2005), 223-239; Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations, Acta Math. Sinica, 23 (2007) 2205-2212] and Ran and Reurings [A fixed point theorem in partially ordered sets and some applications to matrix equations, Proc. Amer. Math. Soc. 132 (2004), 1435-1443], to more general classes of contractive type mappings in partially ordered metric spaces.
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