A STRONG QUADRATIC FUNCTIONAL EQUATION IN C^*-ALGEBRAS

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Abstract. In this paper, we use a fixed point method to investigate the problem of stability on C^*-algebras of the strong quadratic functional equation

$$f(x) + f(y) = f(\sqrt{xx^* + yy^*}).$$

Key Words and Phrases: Generalized Hyers-Ulam stability, quadratic function, C^*-algebra, generalized metric space, fixed point.
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