

SOME RESULTS ON ASYMPTOTICALLY PSEUDOCONTRACTIVE MAPPINGS

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Abstract. Let K be a nonempty closed convex subset of a real Banach space E , $T : K \rightarrow K$ a uniformly continuous asymptotically pseudocontractive mapping having $T(K)$ bounded with sequence $\{k_n\}_{n \geq 0} \subset [1, \infty)$, $\lim_{n \rightarrow \infty} k_n = 1$ such that $p \in F(T) = \{x \in K : Tx = x\}$. Let $\{\alpha_n\}_{n \geq 0}, \{\beta_n\}_{n \geq 0} \in [0, 1]$ be such that $\sum_{n \geq 0} \alpha_n^2 = \infty$ and $\lim_{n \rightarrow \infty} \alpha_n = 0 = \lim_{n \rightarrow \infty} \beta_n$. For arbitrary $x_0 \in K$ let $\{x_n\}_{n \geq 0}$ be iteratively defined by

$$\begin{aligned}x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n T^n y_n, \\y_n &= (1 - \beta_n)x_n + \beta_n T^n x_n, \quad n \geq 0.\end{aligned}$$

Then $\{x_n\}_{n \geq 0}$ converges strongly to $p \in F(T)$.

Key Words and Phrases: Modified two-step iterative scheme, uniformly continuous mappings, uniformly L -Lipschitzian mappings, asymptotically pseudocontractive mappings, Banach spaces.

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