

GLOBAL BIFURCATION FOR NEUMANN PROBLEMS INVOLVING NONHOMOGENEOUS OPERATORS

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Dedicated to Wataru Takahashi on the occasion of his retirement

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Abstract. We consider a Neumann problem involving nonhomogeneous operators

$$-\operatorname{div}(\Psi(x, \nabla u)) + \Phi(x, u) = \mu |u|^{p-2} u + f(\lambda, x, u, \nabla u) \quad \text{in } \Omega$$

when Ψ , Φ , and f satisfy certain conditions and μ is not an eigenvalue in some sense. The aim of this paper is to study the structure of the set of solutions for the above equation, by applying a bifurcation result for nonlinear equations and a nonlinear spectral theory for homogeneous operators.

Key Words and Phrases: Bifurcation, Neumann problem, nonhomogeneous operators, p -Laplacian.

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