

SHRINKING PROJECTION ALGORITHM FOR FIXED POINTS OF FIRMLY NONEXPANSIVE MAPPINGS AND ITS APPLICATIONS

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Dedicated to Wataru Takahashi on the occasion of his retirement

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Abstract. The purpose of this paper is to study the shrinking projection method for finding common fixed points of firmly nonexpansive mappings. Some strong convergence theorems are proved. The main convergence theorem is also applied to the equilibrium and optimization problems. The results of this paper improve and extend the results of Koji Aoyama, Fumiaki Kohsaka, Wataru Takahashi [Koji Aoyama, Fumiaki Kohsaka, Wataru Takahashi, Shrinking projection methods for firmly nonexpansive mappings, *Nonlinear Analysis* (2009), **71**(2009), 1626-1632] in the following respects: (1) the main convergence theorem has been proved by using the new method; (2) the condition of family of firmly nonexpansive mappings $\{T_n\}_{n=1}^{\infty}$ has been relaxed from the condition (Z) to uniformly closed; (3) the application has been given to find the solution of equilibrium and optimization problems.

Key Words and Phrases: Shrinking projection method, firmly nonexpansive mapping, uniformly closed, equilibrium problem.

2010 Mathematics Subject Classification: 47H05, 47H09, 47H10.

Acknowledgment. This project is supported by the National Natural Science Foundation of China under grant (11071279).

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The paper was presented at The 9th International Conference on Fixed Point Theory and Its Applications, July 16-22, 2009, National Changhua University of Education, Changhua, Taiwan (R.O.C.).

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Received: December 31, 2009; Accepted: May 2, 2010.