CONVERGENCE OF VISCOSITY ITERATIVE SCHEMES FOR NONEXPANSIVE SEMIGROUPS

JONG SOO JUNG

Dedicated to Wataru Takahashi on the occasion of his retirement

Department of Mathematics, Donog-A University, Busan 604-714, Korea
E-mail: jungjs@mail.donga.ac.kr

Abstract. Let $E$ be a reflexive Banach space having a weakly sequentially continuous duality mapping $J_\varphi$ with a gauge function $\varphi$, $C$ a nonempty closed convex subset of $E$, $f : C \to C$ a contraction, and $\{T(t) : t \geq 0\}$ a nonexpansive semigroup on $C$ with the fixed point set $F := \bigcap_{t \geq 0} F(T(t)) \neq \emptyset$. Strong convergence theorems of the following implicit and explicit viscosity iterative schemes are established:

$$x_t = \lambda_t f(x_t) + (1 - \lambda_t) T(t)x_t, \quad t \in (0, \infty)$$

where $\{\lambda_t\}_{t \in (0, \infty)}$ is a net in $(0, 1)$ such that $\lim_{t \to \infty} \lambda_t = 0$, and

$$x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n) T(t_n)x_n, \quad n \geq 0,$$

where $\{\alpha_n\} \subset (0, 1)$ and $\{t_n\} \subset \mathbb{R}^+$. The limit point is the unique solution of a certain variational inequality.

Key Words and Phrases: Viscosity iterative scheme, nonexpansive semigroups, common fixed point, contraction, weakly sequentially continuous duality mapping, variational inequality.

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REFERENCES


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