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CONVERGENCE OF VISCOSITY ITERATIVE SCHEMES FOR NONEXPANSIVE SEMIGROUPS

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Dedicated to Wataru Takahashi on the occasion of his retirement

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Abstract. Let E be a reflexive Banach space having a weakly sequentially continuous duality mapping J_φ with a gauge function φ, C a nonempty closed convex subset of E, $f : C \to C$ a contraction, and $\{T(t) : t \geq 0\}$ a nonexpansive semigroup on C with the fixed point set $F := \bigcap_{t\geq 0} F(T(t)) \neq \emptyset$. Strong convergence theorems of the following implicit and explicit viscosity iterative schemes are established:

$$x_t = \lambda_t f(x_t) + (1 - \lambda_t)T(t)x_t, \quad t \in (0, \infty)$$

where $\{\lambda_t\}_{t\in(0,\infty)}$ is a net in $(0, 1)$ such that $\lim_{t\to\infty} \lambda_t = 0$, and

$$x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n)T(t_n)x_n, \quad n \geq 0,$$

where $\{\alpha_n\} \subset (0, 1)$ and $\{t_n\} \subset \mathbb{R}^+$. The limit point is the unique solution of a certain variational inequality.

Key Words and Phrases: Viscosity iterative scheme, nonexpansive semigroups, common fixed point, contraction, weakly sequentially continuous duality mapping, variational inequality.
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