

CONVERGENCE THEOREMS FOR NONEXPANSIVE MAPPINGS AND INVERSE-STRONGLY MONOTONE MAPPINGS

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Abstract. In this paper, we introduce a general iterative scheme for finding a common element of the set of common fixed points of an infinite family of nonexpansive mappings and the set of solutions of variational inequalities for an inverse-strongly monotone mapping.

Key Words and Phrases: Projection, inverse-strongly monotone mapping, nonexpansive mapping, common fixed point, Hilbert space.

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