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THE STABILITY OF A QUARTIC TYPE FUNCTIONAL EQUATION WITH THE FIXED POINT ALTERNATIVE

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Abstract. Cădariu and Radu [Fixed points and the stability of Jensen's functional equation, *J. Inequal. Pure Appl. Math.* **4** (2003), Art. ID 4.] applied the fixed point alternative to the investigation of Cauchy and Jensen functional equations. In this paper, we adopt the the fixed point alternative method of Cădariu and Radu to prove the generalized Hyers-Ulam stability for the quartic functional equation

$$f(kx+y)+f(kx-y)=k^2[f(x+y)+f(x-y)]+2k^2(k^2-1)f(x)-2(k^2-1)f(y)$$

for each $k \in \mathbb{N} \setminus \{1\}$.
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