

AN OPTIMIZATION PROBLEM RELATED TO GENERALIZED EQUILIBRIUM AND FIXED POINT PROBLEMS WITH APPLICATIONS

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Dedicated to Wataru Takahashi on the occasion of his retirement

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Abstract. We introduce a method for finding a solution of an optimization problem to the set of common solutions of generalized equilibrium problems and the set of common fixed points of a family of infinite nonexpansive mappings. The results presented in this paper improve and extend the corresponding results of many others.

Key Words and Phrases: Optimization problem, generalized equilibrium problem, fixed point problem, convex feasibility problem, nonexpansive mapping, W -mapping.

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