

## AN EXPLICIT ITERATIVE METHOD FOR FINDING A COMMON SOLUTION OF EQUILIBRIUM AND FIXED POINT PROBLEMS

RUDONG CHEN\*, QINWEI FAN\* AND HONG-KUN XU\*\*,†

*Dedicated to Wataru Takahashi on the occasion of his retirement*

\*Department of Mathematics, Tianjin Polytechnic University  
Tianjin 300160, China  
E-mails: tjcrd@yahoo.com.cn; fanqinwei2008@yahoo.com.cn

\*\*Department of Applied Mathematics, National Sun Yat-sen University  
Kaohsiung 80424, Taiwan  
and  
Department of Mathematics, College of Science  
King Saud University, Riyadh 11451, Saudi Arabia  
E-mail: xuhk@math.nsysu.edu.tw

**Abstract.** We provide an iterative method for finding a common solution of a finite family of equilibrium problems and of a fixed point problem, and prove its strong convergence. Our method extends an implicit method of Colao, et al. (*Nonlinear Anal.* 71 (2009), no. 7-8, 2708-2715) to an explicit method (in the case of a single nonexpansive mapping).

**Key Words and Phrases:** Equilibrium problem, fixed point, iterative method, nonexpansive mapping, projection, variational inequality.

**2010 Mathematics Subject Classification:** 47H09, 47H10, 47J20, 49J30.

### REFERENCES

- [1] A. Aleyner, S. Reich, *Approximating common fixed points of nonexpansive mappings in Banach spaces*, *Fixed Point Theory*, **10**(2009), No. 1, 3-17.
- [2] M. Bianchi, S. Schaible, *Generalized monotone bifunctions and equilibrium problems*, *J. Optim. Theory Appl.*, **90**(1996), 31-43.
- [3] E. Blum, W. Oettli, *From optimization and variational inequalities to equilibrium problems*, *Math. Student*, **63**(1994), 123-145.
- [4] L. C. Ceng, A. Petruşel, J. C. Yao, *Weak convergence theorem by a modified extragradient method for nonexpansive mappings and monotone mappings*, *Fixed Point Theory*, **9**(2008), No. 1, 73-87.
- [5] L. C. Ceng, J. C. Yao, *Hybrid viscosity approximation schemes for equilibrium problem and fixed point problems of infinitely many nonexpansive mappings*, *Appl. Math. Comp.*, **198**(2008), 729-741.
- [6] F. Cianciaruso, G. Marino, L. Muglia, *Ishikawa iterations for equilibrium and fixed point problems for nonexpansive mappings in Hilbert spaces*, *Fixed Point Theory*, **9**(2008), No. 2, 449-464.

---

†Corresponding author.

- [7] V. Colao, G. Marino, H.K. Xu, *An iterative method for finding common solutions of equilibrium and fixed point problems*, J. Math. Anal. Appl., **344**(2008), 340352.
- [8] V. Colao, G. Lopez-Acedo, G. Marino, *An implicit method for finding common solutions of variational inequalities and systems of equilibrium problems and fixed points of infinite family of nonexpansive mappings*, Nonlinear Anal., **71** (2009), No. 7-8, 2708-2715.
- [9] P.L. Combettes, S.A. Hirstoaga, *Equilibrium programming in Hilbert spaces*, J. Nonlinear Convex Anal., **6** (2005), 117-136.
- [10] Y.L. Cui, X. Liu, *Notes on Browder's and Halpern's methods for nonexpansive mappings*, Fixed Point Theory, **10**(2009), No. 1, 89-98.
- [11] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory*, in: Cambridge Studies in Advanced Mathematics, vol. 28, Cambridge University Press, Cambridge, 1990.
- [12] S. He, H.K. Xu, *Variational Inequalities Governed by Boundedly Lipschitzian and Strongly Monotone Operators*, Fixed Point Theory, **10**(2009), No. 2, 245-258.
- [13] A.N. Iusem, W. Sosa, *New existence results for equilibrium problems*, Nonlinear Anal., **52**(2003), No. 2, 621-635.
- [14] A.N. Iusem, W. Sosa, *Iterative algorithms for equilibrium problems*, Optimization, **52**(2003), No. 3, 301-316.
- [15] G. Lopez, V. Martin-Marquez, H.K. Xu, *Halpern's iteration for nonexpansive mappings*, in "Nonlinear Analysis and Optimization I: Nonlinear Analysis," Contemporary Mathematics, vol. 513, 2010, 211-230.
- [16] G. Marino, H.K. Xu, *A general iterative method for nonexpansive mappings in Hilbert spaces*, J. Math. Anal. Appl., **318** (2006), 43-52.
- [17] J.W. Peng, J.C. Yao, *A modified CQ method for equilibrium problems, fixed points and variational inequality*, Fixed Point Theory, **9**(2008), No. 2, 515-531.
- [18] S. Plubtieng, R. Punpaeng, *A general iterative method for equilibrium problems and fixed point problems in Hilbert spaces*, J. Math. Anal. Appl., **336**(2007), 455-469.
- [19] K. Shimoji, W. Takahashi, *Strong convergence to common fixed points of infinite nonexpansive mappings and applications*, Taiwanese J. Math., **5**(2001), 387-404.
- [20] S. Takahashi, W. Takahashi, *Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces*, J. Math. Anal. Appl., **331**(2007), 506-515.
- [21] H.K. Xu, *Iterative algorithms for nonlinear operators*, J. London Math. Soc., **66**(2002), 240-256.
- [22] H.K. Xu, *An iterative approach to quadratic optimization*, J. Optim. Theory Appl., **116** (2003), 659-678.
- [23] H.K. Xu, *An alternative regularization method for nonexpansive mappings with applications*, in "Nonlinear Analysis and Optimization I: Nonlinear Analysis," Contemporary Mathematics, vol. 513, 2010, 239-263.
- [24] Y. Yao, Y.C. Liou, C. Lee, M.M. Wong, *Convergence Theorem for Equilibrium Problems and Fixed Point Problems*, Fixed Point Theory, **10**(2009), No. 2, 347-363.

*Received: December 31, 2009; Accepted: May 2, 2010.*