

A HYBRID EXTRAGRADIENT METHOD FOR ASYMPTOTICALLY STRICT PSEUDO-CONTRACTIONS IN THE INTERMEDIATE SENSE

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Dedicated to Wataru Takahashi on the occasion of his retirement

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Abstract. In this paper we construct a new hybrid extragradient method for finding a common element of the fixed point set of an asymptotically strict pseudo-contraction in the intermediate sense and the solution set of the variational inequality for an inverse-strongly monotone mapping in a Hilbert space. A strong convergence theorem of the proposed method is established and some of its special cases are also discussed.

Key Words and Phrases: Hybrid extragradient method, modified Mann iteration, variational inequality, strict pseudo-contraction, asymptotically strict pseudo-contraction in the intermediate sense, inverse-strongly monotone mapping, demiclosedness principle.

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