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APPROXIMATING COMMON FIXED POINTS OF NONCOMMUTING ALMOST CONTRACTIONS IN METRIC SPACES

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Abstract. In this paper we prove the existence of coincidence points and common fixed points of noncommuting almost contractions in metric spaces. Moreover, a method for approximating the coincidence points or the common fixed points is also constructed, for which both *a priori* and *a posteriori* error estimates are obtained. These results generalize, extend and unify several classical and very recent related results in literature.
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1. Introduction

Many of the most important nonlinear problems of applied mathematics reduce to solving a given equation which in turn may be reduced to finding the fixed points of a certain mapping or the common fixed points of two mappings. This explains why the study of fixed and common fixed points of mapping satisfying certain contractive conditions attracted more researchers and stimulated an impressive research work in the last three decades, see for example [25] and the very recent monograph [26].

Among these (common) fixed point theorems, only a few are important from a practical point of view, that is, they provide a *constructive* method for finding the fixed points or the common fixed points of the mappings involved, and only seldom they offer information on the error estimate (or rate of convergence) of that iterative method used.

But, from a practical point of view it is important not only to know that the (common) fixed point exists (and, possibly, is unique), but also to be able to construct that (common) fixed point.

In a very recent paper [11], we obtained existence results of coincidence and common fixed points for a class of noncommuting discontinuous contractive mappings which generalize, extend and unify the results in [1] and in some other related papers, and also provide an iterative method for approximating these points. A priori and a posteriori error estimates, expressed by a unique formula, as well the rate of convergence for this method, were also obtained.

As the Zamfirescu fixed point theorem was extended to the class of almost contraction - a large class of contractive type mappings introduced in [6] and also studied in [3], [2] and in many other papers, see [9] and the bibliography therein - the main aim of the present paper is to extend the results in [1] and [11] to almost contractions.

2. Preliminaries

The classical contraction mapping principle is one of the most useful results in fixed point theory. In a metric space setting its statement is given by the next theorem.

Theorem 2.1. *Let (X, d) be a complete metric space and $T : X \longrightarrow X$ a map satisfying*

$$d(Tx, Ty) \leq a\, d(x, y)\,, \quad \text{for all}\, x, y \in X\,, \tag{2.1}$$

where $0 \leq a < 1$ is constant. Then:

$(p1)$ T has a unique fixed point x^* in X;

$(p2)$ The Picard iteration $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n\,, \quad n = 0, 1, 2, \dots \tag{2.2}$$

converges to x^*, for any $x_0 \in X$.

$(p3)$ The following estimate holds:

$$d(x_{n+i-1}, x^*) \leq \frac{a^i}{1-a}\, d(x_n, x_{n-1})\,, \quad n = 0, 1, 2, \dots ; i = 1, 2, \dots \tag{2.3}$$

$(p4)$ The rate of convergence of Picard iteration is given by

$$d(x_n, x^*) \leq a\, d(x_{n-1}, x^*)\,, \quad n = 1, 2, \dots \tag{2.4}$$

Remark 2.2. Theorem 2.1 has many applications in solving nonlinear equations. Its merit is not only to state the existence and uniqueness of the fixed point of the strict contraction T but also to show that the fixed point can be approximated by means of Picard iteration (2.2). Moreover, for this iterative method both *a priori*

$$d(x_n, x^*) \leq \frac{a^n}{1-a}\, d(x_0, x_1)\,, \quad n = 0, 1, 2, \dots$$

and *a posteriori*

$$d(x_n, x^*) \leq \frac{a}{1-a}\, d(x_n, x_{n-1})\,, \quad n = 0, 1, 2, \dots ; i = 1, 2, \dots$$

error estimates are available, which are both obtained from (2.3). On the other hand, the inequality (2.4) shows that the rate of convergence of Picard iteration is linear.

Despite these important features, Theorem 2.1 suffers from one drawback - the contractive condition (2.1) forces T be continuous on X.

It was then natural to ask if there exist or not weaker contractive conditions which do not imply the continuity of T. This was answered in the affirmative by R. Kannan [19] in 1968, who proved a fixed point theorem which extends Theorem 2.1 to mappings that need not be continuous on X (but are continuous at their fixed point), see [23], by considering instead of (2.1) the next condition: there exists $b \in \left[0, \frac{1}{2}\right)$ such that

$$d(Tx, Ty) \leq b\big[d(x, Tx) + d(y, Ty)\big], \quad \text{for all } x, y \in X\,. \tag{2.5}$$

Following the Kannan's theorem, a lot of papers were devoted to obtaining fixed point or common fixed points theorems for various classes of contractive type conditions that do not require the continuity of T, see for example, [24], [25], [9] and the references therein.

One of them, actually a sort of dual of Kannan fixed point theorem, due to Chatterjea [14], is based on a condition similar to (2.5): there exists $c \in \left[0, \frac{1}{2}\right)$ such that

$$d(Tx, Ty) \leq c\big[d(x, Ty) + d(y, Tx)\big], \quad \text{for all } x, y \in X. \tag{2.6}$$

For a presentation and comparison of such kind of fixed point theorems, see [21], [22], [20] and [9].

On the other hand, in 1972, Zamfirescu [28] obtained a very interesting fixed point theorem which gather together all three contractive conditions mentioned above, i.e., (2.1) of Banach, (2.5) of Kannan and (2.6) of Chatterjea, in a rather unexpected way: if T is such that, for any $x, y \in X$, at least one of the conditions (2.1), (2.5) and (2.6) holds, then T has a unique fixed point. Note that considering conditions (2.1), (2.5) and (2.6) all together is not trivial since, as shown later by Rhoades [21], the contractive conditions (2.1) and (2.5), as well as (2.1) and (2.6), respectively, are independent.

These fixed point results were then complemented by coresponding results regarding the existence of common fixed points. So, Jungck [16] proved in 1976 a common fixed point theorem for commuting maps, thus generalizing Theorem 2.1. In the same spirit, very recently M. Abbas and G. Jungck [1], obtained coincidence and common fixed point theorems for the class of Banach contractions, Kannan contractions and Chatterjea contractions, respectively, in cone metric spaces, without making use of the commutative property, but based on the so called concept of weakly compatible mappings, introduced by Jungck [17].

A common fixed point version of Zamfirescu's fixed point theorem, including also the error and rate of convergence estimates, similar to that given in the very recent paper [10], was obtained in the recent paper [11].

The Zamfirescu fixed point theorem has been further extended to almost contractions, a class of contractive type mappings which exhibits totally different features than the ones of the particular results incorporated, i.e., any almost contraction does not have generally a unique fixed point, see Example 1 in [6].

We give here the full statement of the main result from [6] in view of its extension to coincidence and common fixed point theorems.

Theorem 2.3. *Let (X, d) be a complete metric space and $T : X \to X$ an almost contraction, that is a mapping for which there exist a constant $\delta \in (0, 1)$ and some*

$L \geq 0$ such that

$$d(Tx, Ty) \leq \delta \cdot d(x, y) + Ld(y, Tx)\,, \quad \textit{for all } x, y \in X\,. \tag{2.7}$$

Then

1) *$F(T) = \{x \in X : Tx = x\} \neq \phi$;*

2) *For any $x_0 \in X$, the Picard iteration $\{x_n\}_{n=0}^{\infty}$ given by* (1.2) *converges to some $x^* \in F(T)$;*

3) *The following estimate holds*

$$d(x_{n+i-1}, x^*) \leq \frac{\delta^i}{1-\delta}\, d(x_n, x_{n-1})\,, \quad n = 0, 1, 2, \ldots;\; i = 1, 2, \ldots \tag{2.8}$$

It is therefore the main aim of this paper to extend and unify all the results in [1], Theorem 2.3 and several other related results in literature, by proving a general result regarding the existence, the uniqueness and the approximation of common fixed points of two discontinuous weakly contractive mappings of Zamfirescu type.

To this end we need some notions and results from [1] and [17].

Definition 2.4. ([1]) Let S and T be selfmaps of a nonempty set X. If there exists $x \in X$ such that $Sx = Tx$ then x is called a *coincidence point* of S and T , while $y = Sx = Tx$ is called a *point of coincidence* of S and T. If $Sx = Tx = x$, then x is called a *common fixed point* of S and T.

Definition 2.5. ([17]) Let S and T be selfmaps of a nonempty set X. The pair of mappings S and T is said to be *weakly compatible* if they commute at their coincidence points.

The next Proposition, which is given in [1] as Proposition 1.4, will be needed to prove the last part in our main results.

Proposition 2.6. *Let S and T be weakly compatible selfmaps of a nonempty set X. If S and T have a unique coincidence point x, then x is the unique common fixed point of S and T.*

For some other recent related results see also [13], [18], [27].

3. Main results

We start this section by presenting a coincidence point theorem.

Theorem 3.1. *Let (X, d) be a metric space and let T, $S : X \to X$ be two mappings for which there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that*

$$d(Tx, Ty) \leq \delta \cdot d(Sx, Sy) + Ld(Sy, Tx)\,, \quad \textit{for all } x, y \in X\,. \tag{3.1}$$

If the range of S contains the range of T and $S(X)$ is a complete subspace of X, then T and S have a coincidence point in X.

Moreover, for any $x_0 \in X$, the iteration $\{Sx_n\}$ defined by (3.3) converges to some coincidence point x^* of T and S, with the following error estimate

$$d(Sx_{n+i-1}, x^*) \leq \frac{\delta^i}{1-\delta}\, d(Sx_n, Sx_{n-1})\,, \quad n = 0, 1, 2, \ldots;\; i = 1, 2, \ldots \tag{3.2}$$

Proof. Let x_0 be an arbitrary point in X. Since $T(X) \subset S(X)$, we can choose a point x_1 in X such that $Tx_0 = Sx_1$ Continuing in this way, for a x_n in X, we can find $x_{n+1} \in X$ such that

$$Sx_{n+1} = Tx_n,\ n = 0, 1, \ldots \tag{3.3}$$

If $x := x_n,\ y := x_{n-1}$ are two successive terms of the sequence defined by (3.3), then by (3.1) we have

$$d(Sx_n, Sx_{n+1}) = d(Tx_{n-1}, Tx_n) \le L \cdot d(Sx_n, Tx_{n-1}) + \delta \cdot d(Sx_{n-1}, Sx_n),$$

which in view of (3.3) yields

$$d(Sx_{n+1}, Sx_n) \le \delta \cdot d(Sx_n, Sx_{n-1}),\ n = 0, 1, 2 \ldots. \tag{3.4}$$

Now by induction, from (3.4) we obtain

$$d(Sx_{n+k}, Sx_{n+k-1}) \le \delta^k \cdot d(Sx_n, Sx_{n-1}),\ n, k = 0, 1, \ldots\ (k \neq 0), \tag{3.5}$$

and then, for $p > i$, we get after straightforward calculations

$$d(Sx_{n+p}, Sx_{n+i-1}) \le \frac{\delta^i(1 - \delta^{p-i+1})}{1 - \delta} \cdot d(Sx_n, Sx_{n-1}),\ n \ge 0;\ i \ge 1. \tag{3.6}$$

Take $i = 1$ (3.6) and the, by an inductive process, we get

$$d(Sx_{n+p}, Sx_n) \le \frac{\delta}{1 - \delta} \cdot d(Sx_n, Sx_{n-1}) \le \frac{\delta^n}{1 - \delta} \cdot d(Sx_1, Sx_0),\ n = 0, 1, 2 \ldots,$$

which shows that $\{Sx_n\}$ is a Cauchy sequence.

Since $S(X)$ is complete, there exists a x^* in $S(X)$ such that

$$\lim_{n \to \infty} Sx_{n+1} = x^*. \tag{3.7}$$

We can find $p \in X$ such that $Sp = x^*$. By (3.3) and (3.4) we further have

$$d(Sx_n, Tp) \le \delta d(Sx_{n-1}, Sp) \le \delta^{n-1} d(Sx_1, Sp),$$

which shows that we also have

$$\lim_{n \to \infty} Sx_n = Tp. \tag{3.8}$$

Now by (3.7) and (3.8) it results now that $Tp = Sp$, that is, p is a coincidence point of T and S (or x^* is a point of coincidence of T and S). The estimate (3.2) is obtained from (3.6) by letting $p \to \infty$. □

Remark 3.2. Let us note that the coincidence point ensured by Theorem 3.1 is not generally unique, see Example 1 in [6].

In order to obtain from the coincidence Theorem 3.1 a common fixed point theorem, we need the uniqueness of the coincidence point, which which could be obtained by imposing an additional contractive condition, similar to (3.1).

Theorem 3.3. *Let (X, d) be a metric space and let T, $S : X \to X$ be two mappings satisfying (3.1) for which there exist a constant $\theta \in (0, 1)$ and some $L_1 \ge 0$ such that*

$$d(Tx, Ty) \le \theta \cdot d(Sx, Sy) + L_1 d(Sx, Tx)\,, \quad \text{for all } x, y \in X\,. \tag{3.9}$$

If the range of S contains the range of T and $S(X)$ is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X.

In both cases, for any $x_0 \in X$, the iteration $\{Sx_n\}$ defined by (3.3) converges to the unique common fixed point (coincidence point) x^* of S and T, with the error estimate (3.2)

The convergence rate of the iteration $\{Sx_n\}$ is given by

$$d(Sx_n, x^*) \leq \theta \cdot d(Sx_{n-1}, x^*), \quad n = 1, 2, \dots \tag{3.10}$$

Proof. By the proof of Theorem 3.1, we have that T and S have at least a point of coincidence. Now let us show that T and S actually have a unique point of coincidence. Assume there exists $q \in X$ such that $Tq = Sq$. Then, by (3.9) we get

$$d(Sq, Sp) = d(Tq, Tp) \leq 2\delta d(Sq, Tq) + \delta d(Sq, Tp) = \delta d(Sq, Sp)$$

which shows that $Sq = Sp = x^*$, that is T and S have a unique point of coincidence, x^*.

Now if T and S are weakly compatible, by Proposition 1 it follows that x^* is their unique common fixed point. The estimate (3.10) is obtained by (3.9) by taking $x = x_n$ and $y = x^*$. □

A stronger but simpler contractive condition that ensures the uniqueness of the coincidence point and which actualy unifies (3.1) and (3.9), has been very recently obtained by Babu et al. [2]. We state in the following the common fixed point theorem corresponding to this fixed point result.

Theorem 3.4. *Let (X, d) be a metric space and let T, $S : X \to X$ be two mappings for which there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that*

$$d(Tx, Ty) \leq \delta \cdot d(Sx, Sy) + L \min\{d(Sx, Tx) + d(Sy, Ty) + $$
$$+ d(Sx, Ty) + d(Sy, Tx)\}, \quad \text{for all } x, y \in X. \tag{3.11}$$

If the range of S contains the range of T and $S(X)$ is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X.

In both cases, for any $x_0 \in X$, the iteration $\{Sx_n\}$ defined by (3.3) converges to the unique common fixed point (coincidence point) x^* of S and T, with the error estimate (3.2) and convergence rate given by (3.10).

Proof. If $x := x_n$, $y := x_{n-1}$ are two successive terms of the sequence defined by (3.3), then by (3.11) we have

$$d(Sx_n, Sx_{n+1}) = d(Tx_{n-1}, Tx_n) \leq \delta \cdot d(Sx_{n-1}, Sx_n) + L \cdot M,$$

where

$$M = \min\{d(Sx_n, Tx_n) + d(Sx_{n-1}, Tx_{n-1}) + d(Sx_n, Tx_{n-1}) + d(Sx_{n-1}, Tx_n)\} = 0$$

since $d(Sx_n, Tx_{n-1}) = 0$. The rest of the proof follows as that of Theorem 3.3. □

4. Particular cases and conclusions

1) If $S = I$ (the identity map on X), then by Theorem 3.1 we obtain the existence fixed point theorem given in [6] for almost contractions (Theorem 1). If $S = I$, then by Theorem 3.3 we obtain the existence and uniqueness fixed point theorem given in [6] for almost contractions (Theorem 2). If $S = I$, then by Theorem 3.4 we obtain the existence and uniqueness fixed point theorem given in [2] for strict almost contractions.

2) If $S = I$ and $L = 0$ in condition (3.1), then by Theorem 3.1 we obtain a result that extends the Jungck's common fixed point theorem [16] from commuting mappings to weakly compatible mappings.

Three of the other particular cases that are obtained from our main results are given in the following as coollaries.

Corollary 4.1. *Let (X, d) be a metric space and let T, $S : X \to X$ be two mappings for which there exist $b \in [0, \frac{1}{2})$ such that, for all $x, y \in X$,*

(z_2) $d(Tx, Ty) \leq b\big[d(Sx, Tx) + d(Sy, Ty)\big]$.

If the range of S contains the range of T and $S(X)$ is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X.

In both cases, the iteration $\{Sx_n\}$ defined by (3.3) converges to the unique (coincidence) common fixed point x^* of S and T, for any $x_0 \in X$, with the following error estimate

$$d(Sx_{n+i-1}, x^*) \leq \frac{\delta^i}{1-\delta}\, d(Sx_n, Sx_{n-1})\,, \quad n, i = 0, 1, 2, \ldots\ (i \neq 0), \tag{4.1}$$

where $\delta = \dfrac{b}{1-b}$.

The convergence rate of the iteration $\{Sx_n\}$ is given by

$$d(Sx_n, x^*) \leq \delta \cdot d(Sx_{n-1}, x^*)\,, \quad n = 1, 2, \ldots \tag{4.2}$$

Proof. By condition (z_2) and triangle rule, we get

$$\begin{aligned} d(Tx, Ty) &\leq b\big[d(x, Tx) + d(y, Ty)\big] \leq \\ &\leq b\Big\{\big[d(x, y) + d(y, Tx)\big] + \big[d(y, Tx) + d(Tx, Ty)\big]\Big\} \end{aligned}$$

which yields

$$(1-b)d(Tx, Ty) \leq bd(x, y) + 2b \cdot d(y, Tx)$$

and which implies

$$d(Tx, Ty) \leq \frac{b}{1-b}\, d(x, y) + \frac{2b}{1-b}\, d(y, Tx)\,, \quad \text{for all } \ x, y \in X.\,,$$

Now, in view of $0 < b < \dfrac{1}{2}$, (3.1) holds with $\delta = \dfrac{b}{1-b}$ and $L = \dfrac{2b}{1-b}$. The uniqueness condition (4.2) follows similarly. To obtain the conclusion apply Theorem 3.3. □

Corollary 4.2. *Let (X, d) be a metric space and let T, $S : X \to X$ be two mappings for which there exist $c \in [0, \frac{1}{2})$ such that, for all $x, y \in X$,*

(z_3) $d(Tx, Ty) \le c\big[d(Sx, Ty) + d(Sy, Tx)\big]$.

If the range of S contains the range of T and $S(X)$ is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X.

In both cases, the iteration $\{Sx_n\}$ defined by (3.3) converges to the unique (coincidence) common fixed point x^* of S and T, for any $x_0 \in X$, with the following error estimate

$$d(Sx_{n+i-1}, x^*) \le \frac{\delta^i}{1-\delta}\, d(Sx_n, Sx_{n-1}), \quad n, i = 0, 1, 2, \dots \ (i \neq 0), \tag{4.3}$$

where $\delta = \dfrac{c}{1-c}$.

The convergence rate of the iteration $\{Sx_n\}$ is given by

$$d(Sx_n, x^*) \le \delta \cdot d(Sx_{n-1}, x^*), \quad n = 1, 2, \dots \tag{4.4}$$

Proof. By condition (z_3) and triangle rule, we get

$$d(Tx, Ty) \le \frac{c}{1-c}\, d(x, y) + \frac{2\,c}{1-c}\, d(y, Tx),$$

which is (3.1), with $\delta = \dfrac{c}{1-c} < 1$ and $L = \dfrac{2\,c}{1-c} \ge 0$.

The uniqueness condition (4.2) follows similarly. Now apply Theorem 3.3 to obtain the conclusion. □

By noting that Banach contraction condition does imply (3.1) (with L=0), by Corollaries 4.1 and 4.2 we obtain the main result in [11].

Corollary 4.3. *Let (X, d) be a metric space and let T, $S : X \to X$ be two mappings for which there exist $a \in [0, 1)$, $b, c \in [0, \frac{1}{2})$ such that for all $x, y \in X$, at least one of the following conditions is true:*

(z_1) $d(Tx, Ty) \le a\, d(Sx, Sy)$;

(z_2) $d(Tx, Ty) \le b\big[d(Sx, Tx) + d(Sy, Ty)\big]$;

(z_3) $d(Tx, Ty) \le c\big[d(Sx, Ty) + d(Sy, Tx)\big]$.

If the range of S contains the range of T and $S(X)$ is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X.

In both cases, the iteration $\{Sx_n\}$ defined by (3.3) converges to the unique (coincidence) common fixed point x^* of S and T, for any $x_0 \in X$, with the following error estimate

$$d(Sx_{n+i-1}, x^*) \le \frac{\delta^i}{1-\delta}\, d(Sx_n, Sx_{n-1}), \quad n = 0, 1, 2, \dots; \ i = 1, 2, \dots$$

where $\delta = \max\left\{a, \dfrac{b}{1-b}, \dfrac{c}{1-c}\right\}$.

The convergence rate of the iteration $\{Sx_n\}$ *is given by*

$$d(Sx_n, x^*) \le \delta \cdot d(Sx_{n-1}, x^*)\,, \quad n = 1, 2, \dots$$

It is important to note that all our results established here are very important from a computational point of view, due to the fact that they offer a method for computing the common fixed points (the coincidence points, respectively). Moreover, for the iterative method thus obtained, we have a priori and a posteriori error estimates, both contained in the unified estimates of the form 3.2). Note that in (2.7) and (3.1) we can have $\delta = 0$, provided that in this case we also have $L = 0$, which ensures that Theorem 2.3 and Theorem 3.1 also include the Banach contraction mapping principle.

Several other results can be obtained as particular cases of our main results, see [7], [12], [14], [19] etc.

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