ITERATES OF CESÀRO OPERATORS, VIA FIXED POINT PRINCIPLE

SZILÁRD ANDRÁS AND IOAN A. RUS

Department of Applied Mathematics
Babeş-Bolyai University, Cluj Napoca, Romania
E-mails: andraszk@yahoo.com
iarus@math.ubbcluj.ro

Abstract. In a paper by F. Galaz Fontes and F.J. Solís (Iterating the Cesàro operators, Proc. Amer. Math. Soc., 136(2008), No. 6,2147-2153) the authors study the iterates of Cesàro operators on some subsets of $s(\mathbb{C})$ $(c(\mathbb{C}), c_0(\mathbb{C}), l^\infty(\mathbb{C}))$, on $(C[0,1], \mathbb{C})$ and on $C([0,\infty[, \mathbb{C})$. In this paper we study the iterates of Cesàro operators on $s(\mathbb{B})$, on $C([0,1], \mathbb{B})$ and on $C([0,\infty[, \mathbb{B})$, where $(\mathbb{B}, \|\cdot\|)$ is a Banach space and $s(\mathbb{B})$ is the set of all sequences with elements in $\mathbb{B}$. We use the contraction principle on a metric space and on a gauge space and we prove the convergence of the sequence of iterates on the whole space (endowed with a weaker topology). Our proofs are suggested by the characterization theorem of weakly Picard operators on an L-space (I.A. Rus, Picard operators and applications, Sci. Math. Jpn., 58(2003), 191-219) and our method can be applied to a more general class of averaging operators.

Key Words and Phrases: Cesàro operators, iterate operators, fixed point, weakly Picard operators.

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