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APPROXIMATION OF COMMON FIXED POINTS AND VARIATIONAL SOLUTIONS FOR ONE-PARAMETER FAMILY OF LIPSCHITZ PSEUDOCONTRACTIONS

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Dedicated to Wataru Takahashi on the occasion of his retirement

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Abstract. Let X be a uniformly convex Banach space with a uniformly Gâteaux differentiable norm, let C be a nonempty closed convex subset of X and let $\mathcal{T}=\{T_t : t \in G\}$ be a one-parameter family of Lipschitz pseudocontractions on C such that each $T_t : C \to X$ satisfies the weakly inward condition. For any contraction $f : C \to C$, it is shown that the path $t \mapsto x_t,\ t \in [0,1)$, in C, denoted by $x_t = \alpha_t T_t x_t + (1-\alpha_t) f(x_t)$ is continuous and strongly converges to a common fixed point of $\mathcal{T}$, which is the unique solution of some variational inequality. On the other hand, if $\mathcal{T}=\{T_t : t \in G\}$ is a family of uniformly Lipschitz pseudocontractive self-mappings on C, it is also shown that the iteration process:

$$x_0 \in C,\ x_{n+1} = \beta_n(\alpha_n T_{r_n} x_n + (1-\alpha_n)x_n) + (1-\beta_n) f(x_n),\ n \geq 0,$$

strongly converges to the common fixed point of $\mathcal{T}$, which is the unique solution of the same variational inequality.

Key Words and Phrases: Viscosity approximation method, fixed point problem, variational inequality, Lipschitz pseudocontraction, strong convergence, smooth and uniformly convex Banach space.

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