

ERGODIC PROPERTIES OF A PARTICULAR AMENABLE SEMIGROUP OF MAPPINGS IN A BANACH SPACE

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Abstract. We prove that if S is an amenable semigroup and $\varphi = \{T_t : t \in S\}$ is a semigroup of mappings on a nonempty weakly compact, convex subset C of a Banach space E , generated by $\{T_t : t \in A \subseteq S\}$, such that for each $t \in A$, T_t is of type (γ) and $D(\overline{\text{co}}F_{1/n}(T_t), F(T_t)) \rightarrow 0$, as $n \rightarrow \infty$, then $F(\varphi)$ of common fixed points of φ is nonempty and there exists a retraction P of type (γ) from C onto $F(\varphi)$, such that $PT_t = T_tP = P$ for each $t \in S$, and $Px \in \overline{\text{co}}\{T_tx : t \in S\}$ for each $x \in C$. The compactness of C concludes such imposed conditions.

Key Words and Phrases: Amenable semigroup, common fixed point, mappings of type (γ) , nonexpansive mapping, nonlinear ergodic theorem, retraction.

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