

REMARKS ON ULAM STABILITY OF THE OPERATORIAL EQUATIONS

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Abstract. In this paper we present four types of Ulam stability for operatorial equations: Ulam-Hyers stability, generalized Ulam-Hyers stability, Ulam-Hyers-Rassias stability and generalized Ulam-Hyers-Rassias stability. The relations of Ulam stability with the c -weakly Picard operators are also studied. Some examples and counterexamples are given.

Key Words and Phrases: Ulam-Hyers stability, Ulam-Hyers-Rassias stability, fixed point equation, coincidence point equation, integral equation, differential equation, difference equation, operatorial inclusions.

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