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APPROXIMATE FIXED POINTS OF NONEXPANSIVE MAPS

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Abstract. A subset K of a Banach space is said to have the approximate fixed point property if $\inf\{\|x - T(x)\| : x \in K\} = 0$ for any nonexpansive mapping $T : K \to K$. This is a brief overview of what is known about the approximate fixed point property. Many questions remain open.

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