

VARIATIONAL INEQUALITIES GOVERNED BY BOUNDEDLY LIPSCHITZIAN AND STRONGLY MONOTONE OPERATORS

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Abstract. Consider the variational inequality $VI(C, F)$ of finding a point $x^* \in C$ satisfying the property $\langle Fx^*, x - x^* \rangle \geq 0$ for all $x \in C$, where C is a nonempty closed convex subset of a real Hilbert space H and $F : C \rightarrow H$ is a nonlinear mapping. If F is boundedly Lipschitzian and strongly monotone, then we prove that $VI(C, F)$ has a unique solution and iterative algorithms can be devised to approximate this solution. In the case where C is the set of fixed points of a nonexpansive mapping, we also invent a hybrid iterative algorithm to approximate the unique solution of $VI(C, F)$.

Key Words and Phrases: Variational inequality, strongly monotone, bounded Lipschitz, projection, iterative algorithm, Hilbert space.

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