ABSTRACT AND CONCRETE GRONWALL LEMMAS

CECILIA CRĂCIUN* AND NICOLAE LUNGU**

*Colfe's School

Horn Park Lane

London, SE12 8AW, United Kingdom

and

Babeş-Bolyai University Cluj-Napoca

Department of Applied Mathematics

Kogălniceanu Street, No. 1, 400084 Cluj-Napoca, Romania.

E-mail: ceciliacraciun@yahoo.com

**Technical University of Cluj-Napoca

Department of Mathematics

C. Daicoviciu 15, Cluj-Napoca, Romania.

E-mail: nlungu@math.utcluj.ro

Abstract. Let (X, d) be a metric space and $A : X \to X$ be an increasing Picard operator. It is well known that if we have $x \leq Ax$ for all $x \in X$, then $x \leq x^*$, where x^* is the unique fixed point of A. In this paper we investigate which concrete Gronwall lemmas can be derived from this abstract result.

Key Words and Phrases: Inequalities, Gronwall inequalities, Bihari inequalities, Wendorff-type inequalities, abstract Gronwall lemma, Picard operators.

2000 Mathematics Subject Classification: 47H10, 54H25, 45G10, 45M10.

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