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APPROXIMATE COMMON FIXED POINTS FOR ONE-PARAMETER FAMILY OF NONEXPANSIVE NONSELF-MAPPINGS

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Abstract. Let $\mathcal{T}$ be a one-parameter family of nonexpansive nonself-mappings on a nonempty closed convex subset of a smooth and uniformly convex Banach space X such that the set of common fixed points is nonempty. In this paper, we suggest and analyze a modified viscosity approximation method for the family $\mathcal{T}$ of nonexpansive nonself-mappings. We also prove that the approximate solution obtained by the proposed method converges strongly to a solution of a variational inequality.

Key Words and Phrases: Viscosity approximation method, fixed point problem, variational inequality, nonexpansive mapping, strong convergence, smooth and uniformly convex Banach space, demiclosedness.

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References

[1] S. Reich, *Product formulas, nonlinear semigroups and accretive operators*, J. Funct. Anal., **36**(1980), 147-168.

[2] F.E. Browder, *Convergence of approximations to fixed points of nonexpansive mappings in Banach spaces*, Arch. Ration. Mech. Anal., **24**(1967), 82-90.

[3] B. Halpern, *Fixed points of nonexpansive maps*, Bull. Amer. Math. Soc., **73**(1967), 957-961.

[4] P.L. Lions, *Approximation de points fixes de contractions*, C.R. Acad. Sci. Paris Ser. A-B, **284**(1977), 1357-1359.

[5] S. Reich, *Strong convergence theorems for resolvents of accretive operators in Banach spaces*, J. Math. Anal. Appl., **75**(1980), 287-292.

[6] T. Suzuki, *Strong convergence of Krasnoselskii and Mann's type sequences for one-parameter nonexpansive semigroups without Bochner integers*, J. Math. Anal. Appl., **305**(2005), 227-239.

[7] R. Wittmann, *Approximation of fixed points of nonexpansive mappings*, Arch. Math., **59**(1992), 486-491.

[8] V. Barbu, Th. Precupanu, *Convexity and Optimization in Banach spaces*, Editura Academiei R.S.R., Bucharest, 1978.

[9] B. Prus, *A characterization of uniform convexity and applications to accretive operators*, Hiroshima J. Math., **11**(1981), 229-234.

[10] H.K. Xu, *Iterative algorithms for nonlinear operators*, J. London Math. Soc., **66**(2002), 240-256.

[11] T. Dominguez Benavides, G. Lopez Acedo, H.K. Xu, *Construction of sunny nonexpansive retractions in Banach spaces*, Bull. Austral. Math. Soc., **66**(2002), 9-16.

[12] J.S. Jung, C. Morales, *The Mann process for perturbed m-accretive operators in Banach spaces*, Nonlinear Anal., **46**(2001), 231-243.

[13] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge Univ. Press, Cambridge, 1990.

[14] L.S. Liu, *Iterative processes with errors for nonlinear strongly accretive mappings in Banach spaces*, J. Math. Anal. Appl., **194**(1995), 114-125.

[15] W. Takahashi, Y. Ueda, *On Reich's strong convergence theorems for resolvents of accretive operators*, J. Math. Anal. Appl., **104**(1984), 546-553.

[16] L.C. Ceng, J.C. Yao, *A hybrid iterative scheme for mixed equilibrium problems and fixed point problems*, J. Comput. Appl. Math., **214**(2008), 186-201.

[17] A. Moudafi, *Viscosity approximation methods for fixed point theorems*, J. Math. Anal. Appl., **241**(2000), 46-55.

[18] H.K. Xu, *Viscosity approximation methods for nonexpansive mappings*, J. Math. Anal. Appl., **298**(2004), 279-291.

[19] A. Aleyner, Y. Censor, *Best approximation to common fixed points of a semigroup of nonexpansive operators*, J. Convex Anal., in press.

[20] Y. Yao, M.A. Noor, *On viscosity iterative methods for variational inequalities*, J. Math. Anal. Appl., **325**(2007), 776-787.

[21] M.M. Day, *Normed Linear Spaces*, 3rd ed. Springer-Verlag, Berlin, 1973.

[22] J.P. Gossez, E. Lami Dozo, *Some geometric properties related to the fixed point theory for nonexpansive mappings*, Pacific J. Math., **40**(1972), 565-573.

[23] J.S. Jung, *Convergence theorems of iterative algorithms for a family of finite nonexpansive mappings*, Taiwan. J. Math., **11**(2007), 883-902.

[24] L.C. Ceng, H.K. Xu, *Strong convergence of a hybrid viscosity approximation method with perturbed mappings for nonexpansive and accretive operators*, Taiwanese J. Math., **11**(2007), 661-682.

[25] L.C. Ceng, H.K. Xu, J.C. Yao, *Strong convergence of an iterative method with perturbed mappings for nonexpansive and accretive operators*, Numer. Funct. Anal. Optim., **29**(2008), 1-22.

[26] L.C. Ceng, H.K. Xu, J.C. Yao, *The viscosity approximation method for asymptotically nonexpansive mappings in Banach spaces*, Nonlinear Anal., **69**(2008), 1402-1412.

[27] S. Huang, *Fixed points of a sequence of asymptotically nonexpansive mappings*, Fixed Point Theory, **9**(2008), 465-485.

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