

THE OVER-RELAXED A -PROXIMAL POINT ALGORITHM AND APPLICATIONS TO NONLINEAR VARIATIONAL INCLUSIONS IN BANACH SPACES

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Abstract. Based on the notion of A -maximal relaxed accretiveness, the convergence analysis of the over-relaxed proximal point algorithm in the context of the approximating the solutions of a class of nonlinear variational inclusions is explored. Moreover, some results on the general firm nonexpansiveness are also investigated. The obtained results are general and application-oriented in nature.

Key Words and Phrases: General firm nonexpansiveness, variational inclusions, maximal relaxed accretive mapping, A -maximal relaxed accretive mapping, relaxed proximal point algorithm, generalized resolvent operator.

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