

AN APPLICATION OF A FIXED POINT THEOREM TO A FUNCTIONAL INEQUALITY

M. S. MOSLEHIAN¹ AND A. NAJATI²

¹Department of Pure Mathematics, Ferdowsi University of Mashhad
P.O. Box 1159, Mashhad 91775, Iran
Center of Excellence in Analysis on Algebraic Structures (CEAAS)
Ferdowsi University of Mashhad, Iran
E-mail: moslehian@ferdowsi.um.ac.ir and moslehian@ams.org

²Department of Mathematics, Faculty of Sciences
University of Mohaghegh Ardabili
Ardabil, Iran
E-mail: a.nejati@yahoo.com

Abstract. We investigate the functional inequality

$$\left\| f\left(\frac{x-y}{2} + z\right) + f\left(\frac{y-z}{2} + x\right) + f\left(\frac{z-x}{2} + y\right) \right\| \leq \|f(x+y+z)\|$$

and use a fixed point method to prove its stability in the setting of Banach modules over a C^* -algebra.

Key Words and Phrases: Generalized metric space, fixed point, stability, Banach module, C^* -algebra.

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