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COMMON FIXED POINT AND APPROXIMATION RESULTS FOR GENERALIZED ϕ-CONTRACTIONS

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Abstract. We establish common fixed point theorems for weakly compatible generalized ϕ-contractions. As applications, various common fixed point and best approximation results for C_q-commuting and compatible maps are derived. Our results unify, extend and complement various known results existing in the literature.

Key Words and Phrases: Common fixed point, weakly compatible maps, comparison function ϕ, C_q-commuting maps.

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References

[1] R.P. Agarwal, D. O'Regan and M. Sambandham, *Random and deterministic fixed point theory for generalized contractive maps,* Appl. Anal., **83**(2004), 711-725.

[2] M.A. Al-Thagafi, *Common fixed points and best approximation*, J. Approx. Theory, **85**(1996), 318-323.

[3] M.A. Al-Thagafi and N. Shahzad, *Noncommuting selfmaps and invariant approximations*, Nonlinear Anal., **64**(2006), 2778-2786.

[4] V. Berinde, *Iterative Approximation of Fixed Points* (Second edition), Lecture Notes in Mathematics, 1912, Springer, Berlin, 2007.

[5] V. Berinde, *A common fixed point theorem for quasi contractive type mappings*, Ann. Univ. Sci. Budapest, **46**(2003), 81-90.

[6] D.W. Boyd and J.S.W. Wong, *On nonlinear contractions*, Proc. Amer. Math. Soc., **20**(1969), 458-464.

[7] A. Carbone, B.E. Rhoades and S.P. Singh, *A fixed point theorem for generalized contraction map*, Indian J. Pure Appl. Math., **20**(1989), 543-548.

[8] Lj.B. Ciric, *A generalization of Banach's contraction principle*, Proc. Amer. Math. Soc., **45**(1974), 267-273.

[9] P.Z. Daffer and H. Kaneko, *Applications of f-contraction mappings to nonlinear integral equations*, Bull. Inst. Math. Acad. Sinica, **22**(1994), 69-74.

[10] N. Hussain, *Common fixed point and invariant approximation results*, Demonstratio Math., **39**(2006), 389-400.

[11] N. Hussain and V. Berinde, *Common fixed point and invariant approximation results in certain metrizable topological vector spaces*, Fixed Point Theory and Appl., vol. 2006, Article ID 23582, 1-13.

[12] N. Hussain and G. Jungck, *Common fixed point and invariant approximation results for noncommuting generalized (f, g)-nonexpansive maps*, J. Math. Anal. Appl., **321**(2006), 851-861.

[13] N. Hussain and A.R. Khan, *Common fixed point results in best approximation theory*, Applied Math. Lett., **16**(2003), 575-580.

[14] N. Hussain, D. O'Regan and R.P. Agarwal, *Common fixed point and invariant approximation results on non-starshaped domains*, Georgian Math. J., **12**(2005), 659-669.

[15] N. Hussain and B.E. Rhoades, *C_q-commuting maps and invariant approximations*, Fixed Point Theory and Appl., **2006**(2006), Article ID 24543, 1-9.

[16] G. Jungck, *Commuting mappings and fixed points*, Amer. Math. Monthly, **83**(1976), 261-263.

[17] G. Jungck, *Common fixed points for commuting and compatible maps on compacta*, Proc. Amer. Math. Soc., **103**(1988), 977-983.

[18] G. Jungck, *Coincidence and fixed points for compatible and relatively nonexpansive maps*, Int. J. Math. Math. Sci., **16**(1993), 95-100.

[19] G. Jungck and N. Hussain, *Compatible maps and invariant approximations*, J. Math. Anal. Appl., **325**(2007), 1003-1012.

[20] G. Jungck and S. Sessa, *Fixed point theorems in best approximation theory*, Math. Japon., **42**(1995), 249-252.

[21] D. O'Regan and N. Shahzad, *Invariant approximations for generalized I-contractions*, Numer. Func. Anal. Optimiz., **26**(2005), 565-575.

[22] D. O'Regan and N. Hussain, *Generalized I-contractions and pointwise R-subweakly commuting maps*, Acta Math. Sinica, **23**(2007), 1505-1508.

[23] R.P. Pant, *Common fixed points of noncommuting mappings*, J. Math. Anal. Appl., **188**(1994), 436-440.

[24] S.A. Sahab, M.S. Khan and S. Sessa, *A result in best approximation theory*, J. Approx. Theory, **55**(1988), 349-351.

[25] N. Shahzad, *Invariant approximations and R-subweakly commuting maps*, J. Math. Anal. Appl., **257**(2001), 39-45.

[26] N. Shahzad, *On R-subweakly commuting maps and invariant approximations in Banach spaces,* Georgian Math. J., **12**(2005), 157-162.

[27] S.P. Singh, *An application of fixed point theorem to approximation theory*, J. Approx. Theory, **25**(1979), 89-90.

[28] S.P. Singh, *Applications of fixed point theorems in approximation theory*, in: V. Lakshmikantham (Ed.), Applied Nonlinear Analysis, Academic Press, New York, 1979, 389-394.

[29] S.P. Singh, B. Watson, and P. Srivastava, *Fixed Point Theory and Best Approximation: The KKM-Map Principle,* Kluwer Academic Publishers, Dordrecht,1997.

[30] A. Smoluk, *Invariant approximations*, Mat. Stos., **17**(1981), 17-22.

[31] P.V. Subrahmanyam, *An application of a fixed point theorem to best approximation*, J. Approx. Theory, **20**(1977), 165-172.

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