

NOTES ON BROWDER'S AND HALPERN'S METHODS FOR NONEXPANSIVE MAPPINGS

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Abstract. We show that a projection applied to Browder's and Halpern's methods can find the minimum-norm fixed point of a nonexpansive mapping. This supplements the results in existing literature on iterative methods for finding fixed points of nonexpansive mappings. An application to finding the minimum-norm solution of a convex minimization problem is included.

Key Words and Phrases: Projection, nonexpansive mapping, fixed point, minimum-norm, Browder's method, Halpern's method, convex minimization.

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