VISCOSITY APPROXIMATION METHODS FOR STRONGLY POSITIVE AND MONOTONE OPERATORS

LU-CHUAN CENG\textsuperscript{1}, ABDUL RAHIM KHAN\textsuperscript{2}, QAMRUL HASAN ANSARI\textsuperscript{3} AND JEN-CHIH YAO\textsuperscript{4},\textsuperscript{*}

\textsuperscript{1}Department of Mathematics, Shanghai Normal University, Shanghai 200234, China
E-mail: zenglc@hotmail.com

\textsuperscript{2}Department of Mathematics and Statistics, King Fahd University of Petroleum & Minerals
P.O. Box 2007, Dhahran 31261, Saudi Arabia
E-mail: arahim@kfupm.edu.sa

\textsuperscript{3}Department of Mathematics and Statistics, King Fahd University of Petroleum & Minerals
P.O. Box 1169, Dhahran 31261, Saudi Arabia;
and Department of Mathematics, Aligarh Muslim University, Aligarh, India
E-mail: qhansari@kfupm.edu.sa

\textsuperscript{4}Department of Applied Mathematics, National Sun Yat-sen University
Kaohsiung, Taiwan 804
E-mail: yaojc@math.nsysu.edu.tw

\textsuperscript{*}Corresponding author

Abstract. In this paper, we suggest and analyze both explicit and implicit iterative schemes for two strongly positive operators and a nonexpansive mapping \( S \) on a Hilbert space. We also study explicit and implicit versions of iterative schemes for an inverse-strongly monotone mapping \( T \) and \( S \) by an extragradient-like approximation method. The viscosity approximation methods are employed to establish strong convergence of the iterative schemes to a common element of the set of fixed points of \( S \) and the set of solutions of the variational inequality for \( T \). As applications, we consider the problem of finding a common fixed point of a nonexpansive mapping and a strictly pseudocontractive mapping which solves some variational inequalities. Our results improve and unify various celebrated results of viscosity approximation methods for fixed-point problems and variational inequality problems.

Key Words and Phrases: General iterative method, viscosity approximation method, hybrid viscosity approximation method, fixed points, inverse-strongly monotone mappings, nonexpansive mappings, variational inequalities, strongly positive operators.

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