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VISCOSITY APPROXIMATION METHODS FOR STRONGLY POSITIVE AND MONOTONE OPERATORS

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Abstract. In this paper, we suggest and analyze both explicit and implicit iterative schemes for two strongly positive operators and a nonexpansive mapping S on a Hilbert space. We also study explicit and implicit versions of iterative schemes for an inverse-strongly monotone mapping T and S by an extragradient-like approximation method. The viscosity approximation methods are employed to establish strong convergence of the iterative schemes to a common element of the set of fixed points of S and the set of solutions of the variational inequality for T. As applications, we consider the problem of finding a common fixed point of a nonexpansive mapping and a strictly pseudocontractive mapping which solves some variational inequalities. Our results improve and unify various celebrated results of viscosity approximation methods for fixed-point problems and variational inequality problems.

Key Words and Phrases: General iterative method, viscosity approximation method, hybrid viscosity approximation method, fixed points, inverse-strongly monotone mappings, nonexpansive mappings, variational inequalities, strongly positive operators.
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