

APPROXIMATING COMMON FIXED POINTS OF NONEXPANSIVE MAPPINGS IN BANACH SPACES

ARKADY ALEYNER¹ AND SIMEON REICH²

¹Department of Mathematics
The Technion - Israel
Institute of Technology
32000 Haifa, Israel
E-mail: aaleyner@tx.technion.ac.il

²Department of Mathematics
The Technion - Israel
Institute of Technology
32000 Haifa, Israel
E-mail: sreich@tx.technion.ac.il

Abstract. Two algorithmic frameworks for finding a common fixed point of a finite collection of nonexpansive mappings in uniformly convex and uniformly smooth Banach spaces, respectively, are proposed. Corresponding weak and strong convergence theorems are established.

Key Words and Phrases: Common fixed point, iteration process, nonexpansive mapping.

2000 Mathematics Subject Classification: 47H09, 47H10, 65J15.

Acknowledgments. The second author was partially supported by the Israel Science Foundation (Grant 647/07), by the Fund for the Promotion of Research at the Technion and by the Technion President's Research Fund.

REFERENCES

- [1] H.H. Bauschke, *The approximation of fixed points of compositions of nonexpansive mappings in Hilbert space*, J. Math. Anal. Appl., **202**(1996), 150-159.
- [2] H.H. Bauschke and J.M. Borwein, *On projection algorithms for solving convex feasibility problems*, SIAM Review, **38**(1996), 367-426.
- [3] R.E. Bruck, *Nonexpansive projections on subsets of Banach spaces*, Pacific J. Math., **47**(1973), 341-355.

- [4] R.E. Bruck, *A simple proof of the mean ergodic theorem for nonlinear contractions in Banach spaces*, Israel J. Math., **32**(1979), 107-116.
- [5] Y. Censor and G.T. Herman, *On some optimization techniques in image reconstruction from projections*, Appl. Numer. Math., **3**(1987), 365-391.
- [6] C.E. Chidume and B. Ali, *Weak and strong convergence theorems for finite families of asymptotically nonexpansive mappings in Banach spaces*, J. Math. Anal. Appl., **330**(2007), 377-387.
- [7] I. Cioranescu, *Geometry of Banach Spaces, Duality Mappings and Nonlinear Problems*, Kluwer, Dordrecht, 1990.
- [8] G. Das and J.P. Debata, *Fixed points of quasi-nonexpansive mappings*, Indian J. Pure Appl. Math., **17**(1986), 1263-1269.
- [9] D. van Dulst, *Equivalent norms and the fixed point property for nonexpansive mappings*, J. London Math. Soc., **25**(1982), 139-144.
- [10] K. Goebel and S. Reich, *Uniform Convexity, Hyperbolic Geometry and Nonexpansive Mappings*, Marcel Dekker, New York and Basel, 1984.
- [11] B. Halpern, *Fixed points of nonexpanding maps*, Bull. Amer. Math. Soc., **73**(1967), 957-961.
- [12] G.T. Herman, *Image Reconstruction from Projections. The Fundamentals of Computerized Tomography*, Academic Press, New York, 1980.
- [13] S. Ishikawa, *Fixed points by a new iteration method*, Proc. Amer. Math. Soc., **44**(1974), 147-150.
- [14] S.H. Khan and H. Fukhar-ud-din, *Weak and strong convergence of a scheme with errors for two nonexpansive mappings*, Nonlinear Anal., **61**(2005), 1295-1301.
- [15] T.-H. Kim and H.-K. Xu, *Strong convergence of modified Mann iterations*, Nonlinear Anal., **61**(2005), 51-60.
- [16] P.-L. Lions, *Approximation de points fixes de contractions*, Comptes Rendus de l'Académie des Sciences Paris, **284**(1977), 1357-1359.
- [17] W.R. Mann, *Mean value methods in iteration*, Proc. Amer. Math. Soc., **4**(1953), 506-510.
- [18] J. O'Hara, P. Pillay and H.-K. Xu, *Iterative approaches to convex feasibility problems in Banach spaces*, Nonlinear Anal., **64**(2006), 2022-2042.
- [19] Z. Opial, *Weak convergence of the sequence of successive approximations for nonexpansive mappings*, Bull. Amer. Math. Soc., **73**(1967), 591-597.
- [20] S. Reich, *Asymptotic behavior of contractions in Banach spaces*, J. Math. Anal. Appl., **44**(1973), 57-70.
- [21] S. Reich, *Weak convergence theorems for nonexpansive mappings in Banach spaces*, J. Math. Anal. Appl., **67**(1979), 274-276.
- [22] S. Reich, *Strong convergence theorems for resolvents of accretive operators in Banach spaces*, J. Math. Anal. Appl., **75**(1980), 287-292.
- [23] S. Reich, *Nonlinear semigroups, accretive operators and applications*, Nonlinear Phenomena in Mathematical Sciences, 831-838, Academic Press, New York, 1982.

- [24] S. Reich, *A limit theorem for projections*, Linear and Multilinear Algebra, **13**(1983), 281-290.
- [25] J. Schu, *Weak and strong convergence of fixed points of asymptotically nonexpansive mappings*, Bull. Austral Math. Soc., **43**(1991), 153-159.
- [26] K.-K. Tan and H.-K. Xu, *Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process*, J. Math. Anal. Appl., **178**(1993), 301-308.
- [27] R. Wittmann, *Approximation of fixed points of nonexpansive mappings*, Archiv der Mathematik, **58**(1992), 486-491.
- [28] H.-K. Xu, *An iterative approach to quadratic optimization*, J. Optimization Theory Appl., **116**(2003), 659-678.

Received: October 6, 2008; Accepted: January 29, 2009.