

## FIXED POINTS FOR MULTIVALUED OPERATORS ON A SET ENDOWED WITH VECTOR-VALUED METRICS AND APPLICATIONS

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**Abstract.** In this paper we present the fixed point theory for multivalued operators on a set endowed with vector valued metrics. Our results extend, to the multivalued case, the result given by A.I. Perov, A.I. Perov and A.V. Kibenko, J. Matkowski, T. Shibata, as well as, some recent work of C. Bacoţiu, D. O'Regan, N. Shahzad and R.P. Agarwal, D. O'Regan and R. Precup, R. Precup, etc.

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