

MINIMAX THEOREMS FOR FUNCTIONS INVOLVING A REAL VARIABLE AND APPLICATIONS

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Abstract. In this paper, we offer an overview of various applications of certain minimax theorems for functions of two variables one of which runs over a real interval.

Key Words and Phrases: Minimax theorems, connectedness, integral functionals on L^p and Sobolev spaces, local and global minima, potential operators, critical points, nonlinear equations, multiplicity, singular set, bifurcation, boundary value problems, well-posedness of constrained minimization problems, Chebyshev sets.

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