

INTEGRAL EQUATIONS, PERIODICITY, AND FIXED POINTS

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Abstract. In this paper we are led to the conjecture that if there is a $T > 0$ with $a(t+T) = a(t)$ and $D(t+T, s+T, x) = D(t, s, x)$ and if D is fairly smooth then the integral equation $x(t) = a(t) + \int_{-\infty}^t D(t, s, x)ds$ has a T -periodic solution. Several results are offered in defense of the conjecture, but the problem is far from being solved. We use Schaefer's fixed point theorem and a variety of Liapunov functionals to prove the results. The most striking feature of the paper is the fact that we can prove that there is a periodic solution either by differentiating D or by integrating D . It is a very attractive problem for further study.

Key Words and Phrases: Integral equations, periodicity, and fixed points.

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REFERENCES

- [1] T.A. Burton, *Liapunov functionals and periodicity in integral equations*, Tohoku Math. J., **46**(1994), 207-220.
- [2] T.A. Burton, *Boundedness and periodicity in integral and integro-differential equations*, Differential Equations and Dynamical Systems, **1**(1993), 161-172.
- [3] T.A. Burton, and Tetsuo Furumochi, *Periodic solutions of Volterra equations and attractivity*, Dynamic Systems and Applications, **3**(1994), 583-598.
- [4] T.A. Burton, and Tetsuo Furumochi, *Periodic solutions of a Volterra equation and robustness*, Nonlinear Analysis, **25**(1995), 1199-1219.
- [5] T.A. Burton, and Tetsuo Furumochi, *Periodic and asymptotically periodic solutions of Volterra integral equations*, Funkcial. Ekvac., **39**(1996), 87-107.

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- [6] T.A. Burton, and Tetsuo Furumochi, *Existence theorems and periodic solutions of neutral integral equations*, Nonlinear Analysis, **43**(2001), 527-546.
- [7] T.A. Burton, and G. Makay, *Continuity, compactness, fixed points, and integral equations*, Electron. J. Qual. Theory Differ. Eq., **14**(2002), 1-13.
- [8] R.K. Miller, *Nonlinear Volterra Integral Equations*, Benjamin, Menlo Park, California, 1971.
- [9] H. Schaefer, *Über die Methode der a priori-Schranken*, Math. Ann., **129**(1955), 415-416.
- [10] D.R. Smart, *Fixed Point Theorems*, Cambridge, 1980.

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