

## FIXED POINTS AND CONTINUITY OF ALMOST CONTRACTIONS

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**Abstract.** Almost contractions form a class of generalized contractions that includes several contractive type mappings like usual contractions, Kannan mappings, Zamfirescu mappings etc. Since any usual contraction is continuous, while a Kannan mapping is not generally continuous but is continuous *at* the fixed point, the main aim of this paper is to study the continuity of both single and multi-valued almost contractions. The main results state that any almost contraction is continuous at its fixed point(s). This answers an open question raised in [Berinde, V., *On the approximation of fixed points of weak contractive mappings* Carpathian J. Math. **19** (2003), No. 1, 7-22].

**Key Words and Phrases:** fixed point, metric space, almost contraction, continuity in the fixed point.

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