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GOOD AND SPECIAL WEAKLY PICARD OPERATOR PROPERTIES FOR THE STANCU OPERATORS

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Abstract. In 1977 Livia d'Apuzzo have introduced the notions of special and good convergence of the sequence of successive approximations on metric spaces. Afterwards, in 2003 I.A.Rus transferred these properties to the operators giving the notions of special and good weakly Picard operators. In this paper are investigated some properties of good and special convergence for some classes of Stancu operators.
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1. Introduction

Let (X, d) be a metric space and $A : X \to X$ an operator. In this paper we shall use the following notations:
$P(X) := \{Y \subset X | \, Y \neq \emptyset\}$;
$F_A := \{x \in X | \, A(x) = x\}$ - the fixed point set of A;
$I(A) := \{Y \in P(X) | \, A(Y) \subset Y\}$ - the family of the nonempty invariant subsets of A;
$A^0 := 1_X, \ A^1 := A, ..., \ A^{n+1} := A \circ A^n, \ n \in \mathbb{N}$.

Definition 1. *(*[5], [6], [8]*) Let (X, d) be a metric space.*
1) An operator $A : X \to X$ is weakly Picard operator (briefly WPO) if the sequence of successive approximations $(A^n(x_0))_{n\in\mathbb{N}}$ converges for all $x_0 \in X$ and the limit (which may depend on x_0) is a fixed point of A.
2) If the operator $A : X \to X$ is WPO and $F_A = \{x^*\}$, then by definition the operator A is Picard operator (briefly PO).

3) If the operator $A : X \to X$ is WPO, then can be considered the operator A^{∞} defined by $A^{\infty} : X \to X,\ A^{\infty}(x) := \lim_{n\to\infty} A^{n}(x)$.

Theorem 1. *(Characterization theorem - [5], [6])An operator $A : X \to X$ is WPO if only if there exits a partition of X, $X = \bigcup_{\lambda\in\Lambda} X_{\lambda}$, such that:*
(a) $X_{\lambda} \in I(A),\ \forall\lambda \in \Lambda$;
(b) $A|_{X_{\lambda}} : X_{\lambda} \to X_{\lambda}$ is PO, $\forall\lambda \in \Lambda$.

In [8], I.A. Rus has introduced the notions of good and special WPO like as:

Definition 2. *([2], [8]) Let (X, d) be a metric space and $A : X \to X$ a WPO.*
1) $A : X \to X$ is good WPO, if the series $\sum_{n=1}^{\infty} d\left(A^{n-1}(x), A^{n}(x)\right)$ converges, for all $x \in X$ (see [8]). In the case that the sequence $\left(d\left(A^{n-1}(x), A^{n}(x)\right)\right)_{n\in\mathbb{N}^*}$ is strictly decreasing for all $x \in X$, the operator A is good WPO of type M (see [2]).
2) *$A : X \to X$ is special WPO, if the series $\sum_{n=1}^{\infty} d(A^{n}(x), A^{\infty}(x))$ converges, for all $x \in X$ (see [8]). When the sequence $(d(A^{n}(x), A^{\infty}(x)))_{n\in\mathbb{N}^*}$ is strictly decreasing for all $x \in X$, the operator A is special WPO of type M (see [2]).*

Theorem 2. *([4]) Let (X, d) be a metric space and $A : X \to X$ a WPO. If A is special WPO then A is good WPO.*

2. Metrical conditions satisfied by Stancu operators

Remark 1. *Generally, an operator of uniform approximation $A : C[a, b] \to C[a, b]$ may satisfy certain contractive conditions such as:*
(1) $\|A(x) - A(y)\| \leqslant \|x - y\|,\ x, y \in C[a, b]$;
(2) $\left\|A^{2}(x) - A(x)\right\| \leqslant L\|x - A(x)\|,\ \forall x \in C[a, b]$, *for* $L \in (0, 1)$;
(3) $C[a, b] = \bigcup_{\gamma\in\mathbb{R}} X_{\gamma}$ *such that:*
(i) X_{γ} is closed and invariant to A for any $\gamma \in \mathbb{R}$;
(ii) $A : X_{\gamma} \to X_{\gamma}$ is α - contraction.

Let $\alpha, \beta \in \mathbb{R},\ 0 \leqslant \alpha \leqslant \beta$ and $n \in \mathbb{N}^*$. We consider the Stancu operators (see [1], [7], [9]):

$$P_{n,\alpha,\beta} : C[0, 1] \to C[0, 1]\ ,\ f \mapsto P_{n,\alpha,\beta}(f)$$

where $P_{n,\alpha,\beta}(f)(x) := \sum\limits_{k=0}^{n} f\left(\frac{k+\alpha}{n+\beta}\right)\begin{pmatrix} n \\ k \end{pmatrix} x^k (1-x)^{n-k}$ The properties as linear positive operators and the properties of uniform approximation of the Stancu operators was obtained and presented in [1], [3], [9], [10]. Let $P^m_{n,\alpha,\beta}$ be the m$^{\text{th}}$ iterate of the operator $P_{n,\alpha,\beta}$. The following results are well-known:

Theorem 3. *([7]) Let $n \in \mathbb{N}^*$ and $\beta > 0$. Then the Stancu operators $P_{n,0,\beta}$ are WPO's on $C\left[0, \frac{n}{n+\beta}\right]$.*

Theorem 4. *([7]) Let $n \in \mathbb{N}^*$ and $\alpha > 0$. Then the Stancu operators $P_{n,\alpha,\alpha}$ are WPO's on $C\left[\frac{\alpha}{n+\alpha}, 1\right]$.*

As in [7] we consider the partition $C\left[0, \frac{n}{n+\beta}\right] = \bigcup\limits_{\gamma\in\mathbb{R}} X_\gamma$ of $C\left[0, \frac{n}{n+\beta}\right]$ where $X_\gamma := \left\{ f \in C\left[0, \frac{n}{n+\beta}\right] \middle| f(0) = \gamma \right\}$, $\gamma \in \mathbb{R}$. X_γ is a closed subset of $C\left[0, \frac{n}{n+\beta}\right]$ and is an invariant subset to $P_{n,0,\beta}$ for all $\beta > 0$, $n \in \mathbb{N}^*$, $\gamma \in \mathbb{R}$. In [7] it is proved that:

$$|P_{n,0,\beta}(f)(x) - P_{n,0,\beta}(g)(x)| \leqslant \left[1 - \left(1 - \frac{n}{n+\beta}\right)^n\right] \|f-g\|_C\,, \qquad (1)$$

$\forall x \in [0,1]$, for all $f, g \in X_\gamma$. So, $P_{n,0,\beta}$ is WPO also in the sense of Theorem 1, where the invariant subsets are in this case X_γ, $\gamma \in \mathbb{R}$. On the other hand, in [7] was proved that:

$$|P_{n,\alpha,\alpha}(f)(x) - P_{n,\alpha,\alpha}(g)(x)| \leqslant \left[1 - \left(1 - \frac{\alpha}{n+\alpha}\right)^n\right] \|f-g\|_C \qquad (2)$$

$\forall x \in [0,1]$, for all $f, g \in X_\gamma$, where

$$X_\gamma = \{f \in C[\frac{\alpha}{n+\alpha}, 1] : f(1) = \gamma\}.$$

Corollary 5. *a) For any $\beta > 0$ the family of Stancu operators*

$$\{P_{n,0,\beta} : n \in \mathbb{N}^*\}$$

has the contractive properties from Remark 1.
b) For any $\alpha \geqslant 0$ the family of Stancu operators $\{P_{n,\alpha,\alpha} : n \in \mathbb{N}^*\}$ has the contractive properties from Remark 1.

It is known that $P_{n,\alpha,\beta}(e_0(x)) = e_0(x) = 1, \forall\, x \in [0,1]\,,\ \forall\, n \in \mathbb{N}^*,\ \forall \alpha,\beta \geqslant 0$ (see [1], [10]) and because the Stancu operators is linear, follows that the fixed points of these operators is $K = \{f \in C[0,1] | \, f - \text{constant}\}$

Theorem 6. *Let $n \in \mathbb{N}^*$ and $\beta > 0$. Then the Stancu operators $P_{n,0,\beta}$ are special WPO and good WPO of type M on $C\left[0, \frac{n}{n+\beta}\right]$.*

Proof. From Theorem 3, $P_{n,0,\beta}$ is WPO.
Let $f \in C[0, \frac{n}{n+\beta}]$. Then $f \in X_{f(0)}$ and according to (1) we infer that $P_{n,0,\beta}$ is contraction on $X_{f(0)}$. Finally, we get that $P_{n,0,\beta}$ is special WPO of type M on $C\left[0, \frac{n}{n+\beta}\right]$. From Theorem 2, any special WPO is good WPO. Then, $P_{n,0,\beta}$ is also good WPO of type M. Moreover, we can see that the sequences

$$\left(d\left(P_{n,0,\beta}^{m}(f), P_{n,0,\beta}^{\infty}(f)\right)\right)_{m\in\mathbb{N}^*} \quad \text{and} \quad \left(d\left(P_{n,0,\beta}^{m}(f), P_{n,0,\beta}^{m-1}(f)\right)\right)_{m\in\mathbb{N}^*}$$

are strictly decreasing for any $f \in C\left[0, \frac{n}{n+\beta}\right] \setminus K$. □

Theorem 7. *Let $n \in \mathbb{N}^*$ and $\alpha > 0$. Then the Stancu operators $P_{n,\alpha,\alpha}$ are special WPO and good WPO of type M on $C\left[\frac{\alpha}{n+\alpha}, 1\right]$.*

Proof. The proof is analogues as in Theorem 6. □

3. Uniform good and special WPO property

Let $\{X_n\}_{n\in\mathbb{N}^*}$ a family of metric spaces such that: $X_1 \supseteq X_2 \supseteq ... \supseteq X_n \supseteq X_{n+1} \supseteq ...$ and for any $n \in \mathbb{N}^*$ consider the operator $A_n : X_n \to X_n$. We will say that the operators $\{A_n : n \in \mathbb{N}^*\}$ are special (good) WPO having uniformity on intersection if A_n is special (good) WPO, $\forall\, n \in \mathbb{N}^*$ and there exist a sequence $(c_n)_{n\in\mathbb{N}^*} \subset \mathbb{R}_+$ (respectively $(d_n)_{n\in\mathbb{N}^*} \subset \mathbb{R}_+$) and a continuous functional

$$\varphi : \bigcap_{n=1}^{\infty} X_n \to \mathbb{R}_+$$

such that:

$$\sum_{m=1}^{\infty} d\left(A_n^m(x), A_n^{\infty}(x)\right) \leqslant c_n \cdot \varphi(x), \ \forall\, x \in \bigcap_{n=1}^{\infty} X_n, \ \forall\, n \in \mathbb{N}^*$$

(respectively, $\sum\limits_{m=1}^{\infty} d\left(A_n^{m-1}(x), A_n^m(x)\right) \leqslant d_n \cdot \varphi(x), \ \forall\, x \in \bigcap\limits_{n=1}^{\infty} X_n, \ \forall\, n \in \mathbb{N}^*$).

In the following, we prove that the Stancu operators have this property.

Theorem 8. *For any $\beta \geqslant 0$ fixed, the family of Stancu operators*

$$\{P_{n,0,\beta} : n \in \mathbb{N}^*\}$$

are special and good WPO having uniformity on intersection.

Proof. Let $\beta \geqslant 0$ fixed. Consider the set

$$B[0,1] = \{f : [0,1] \to \mathbb{R} |\, f - \text{bounded}\}$$

and

$$X = \{f \in B[0,1] |\, f - \text{continuous in } x = 1\}.$$

For any $n \in \mathbb{N}^*$ let $X_n = X \cap C\left[0, \frac{n}{n+\beta}\right]$. It is easy to see that $\bigcap_{n=1}^{\infty} X_n = C[0,1]$. In theorem 6 we have proved that $P_{n,0,\beta}$ are special and good WPO on X_n for each $n \in \mathbb{N}^*$. Using the inequality (1) we obtain the estimation:

$$\begin{aligned}\left|P^1_{n,0,\beta}(f)(x) - f^*(x)\right| &= \left|P^1_{n,0,\beta}(f)(x) - P^1_{n,0,\beta}(f^*)(x)\right| \leqslant \\ &\leqslant \left[1 - \left(1 - \frac{n}{n+\beta}\right)^n\right] \cdot |f(x) - f^*(x)| \\ &= \left[1 - \left(1 - \frac{n}{n+\beta}\right)^n\right] \cdot |f(x) - f(0)| \leqslant \\ &\leqslant \left[1 - \left(1 - \frac{n}{n+\beta}\right)^n\right] \cdot a,\end{aligned}$$

where: $a = diam\,(f([0,1])) = \max\{|f(x) - f(y)| : x, y \in [0,1]\}$ By induction, for $m \in \mathbb{N}^*$, follows:

$$\left|P^m_{n,0,\beta}(f)(x) - f^*(x)\right| \leqslant \left[1 - \left(1 - \frac{n}{n+\beta}\right)^n\right]^m \cdot a$$

for all $x \in [0,1]$. Consequently,

$$\sum_{m=1}^{\infty} \left|P^m_{n,0,\beta}(f)(x) - f^*(x)\right| \leqslant a \cdot \frac{1 - \left(1 - \frac{n}{n+\beta}\right)^n}{\left(1 - \frac{n}{n+\beta}\right)^n}.$$

On the other hand,

$$\left|P^1_{n,0,\beta}(f)(x) - P^0_{n,0,\beta}(f)(x)\right| = \left|\sum_{k=0}^{n} P_{n,k}(x) f\left(\frac{k}{n+\beta}\right) - f(x)\right| =$$

$$= \left|\sum_{k=0}^{n} P_{n,k}(x)\left[f\left(\frac{k}{n+\beta}\right) - f(x)\right]\right| \leqslant a \cdot \sum_{k=0}^{n} P_{n,k}(x) = a,$$

where $a = diam\,(f([0,1]))$. By induction, follows:

$$\begin{aligned}&\left|P^m_{n,0,\beta}(f)(x) - P^{m-1}_{n,0,\beta}(f)(x)\right| \\ &= \left|P^1_{n,0,\beta}\left(P^{m-1}_{n,0,\beta}(f)(x)\right) - P^1_{n,0,\beta}\left(P^{m-2}_{n,0,\beta}(f)(x)\right)\right|\end{aligned}$$

$$\leq ... \leqslant \left[1-\left(1-\frac{n}{n+\beta}\right)^{n}\right]^{m-1}\cdot a.$$

Thus,

$$\sum_{m=1}^{\infty}\left|P_{n,0,\beta}^{m}\left(f\right)\left(x\right)-P_{n,0,\beta}^{m-1}\left(f\right)\left(x\right)\right|\leqslant$$

$$\leqslant \lim_{m\to\infty} a\cdot\left\{1+\left[1-\left(1-\frac{n}{n+\beta}\right)^{n}\right]+...+\left[1-\left(1-\frac{n}{n+\beta}\right)^{n}\right]^{m-1}\right\}=$$

$$=a\cdot\frac{1}{1-\left[1-\left(1-\frac{n}{n+\beta}\right)^{n}\right]}=\frac{a}{\left(1-\frac{n}{n+\beta}\right)^{n}}\ ,\ \forall\ x\in\left[0,1\right],\quad f\in C\left[0,\frac{n}{n+\beta}\right].$$

Let $\varphi : C\left[0,1\right]\to\mathbb{R}_{+},\ \varphi\left(f\right)=diam\left(f\left(\left[0,1\right]\right)\right)=diam\left(\operatorname{Im}f\right)$. From above follows that:

$$\sum_{m=1}^{\infty}d_{c}\left(P_{n,0,\beta}^{m}\left(f\right),f^{*}\right)\leqslant\frac{1-\left(1-\frac{n}{n+\beta}\right)^{n}}{\left(1-\frac{n}{n+\beta}\right)^{n}}\cdot\varphi\left(f\right),\ \forall\ f\in C\left[0,1\right],\ \forall\ n\in\mathbb{N}^{*}$$

and

$$\sum_{m=1}^{\infty}d_{c}\left(P_{n,0,\beta}^{m-1}\left(f\right),P_{n,0,\beta}^{m}\left(f\right)\right)\leqslant\frac{1}{\left(1-\frac{n}{n+\beta}\right)^{n}}\cdot\varphi\left(f\right),\ \forall\ f\in C\left[0,1\right],\ \forall\ n\in\mathbb{N}^{*}$$

It is easy to prove that φ is seminorm on $C\left[0,1\right]$ and $\varphi\left(f-g\right)\leqslant 2\left\|f-g\right\|_{C}$. Since

$$\left|\varphi\left(f\right)-\varphi\left(g\right)\right|\leqslant\varphi\left(f-g\right),\ \forall\ f,g\in C\left[0,1\right],$$

we infer that φ is continuous. Considering

$$c_{n}=\frac{1-\left(1-\frac{n}{n+\beta}\right)^{n}}{\left(1-\frac{n}{n+\beta}\right)^{n}},\qquad d_{n}=\frac{1}{\left(1-\frac{n}{n+\beta}\right)^{n}}$$

we complete the proof. □

Remark 2. *Similar consideration holds for the Stancu operators*

$$\left\{P_{n,\alpha,\alpha}:n\in\mathbb{N}^{*}\right\},$$

for any $\alpha\geqslant 0$ *fixed.*

Remark 3. *Any contraction on a complete metric space is PO and special WPO, but the family of the contractions is not uniform special WPO, nor uniform good WPO. The character of uniformity from the above property is given by the fact that the functional φ is the same for all operators $A_n,\ n \in \mathbb{N}^*$, depending only by the family of operators.*

Let X be an ordered metric space, $A : X \to X$ operator and $Y \subset X$ closed subset (not necessary invariant). If we suppose that A is WPO and if we have:

$$x \leqslant A(x) \leqslant ... \leqslant A^m(x) \leqslant A^{m+1}(x) \leqslant ... \leqslant A^\infty(x),$$

or

$$A^\infty(x) \leq ... \leq A^{m+1}(x) \leqslant A^m(x) \leqslant ... \leqslant A(x) \leqslant x,$$

for all $x \in Y$, then we say that the operator A have monotone iteration on Y.

Proposition 9. *a) For $\alpha \geqslant 0$ fixed, on the set*
$Y = \left\{ f : \left[\frac{\alpha}{n+\alpha}, 1\right] \to \mathbb{R} \middle| f\text{−increasing and convex} \right\}$
the family $\{P_{n,\alpha,\alpha} : n \in \mathbb{N}^*\}$ of Stancu operators have monotone iteration.
b) For $\beta > 0$ fixed, on the set
$Y = \left\{ f : \left[0, \frac{n}{n+\beta}\right] \to \mathbb{R} \middle| f\text{−increasing and concave} \right\}$
the family $\{P_{n,0,\beta} : n \in \mathbb{N}^*\}$ of Stancu operators have monotone iteration.

Proof. a) In this case, $P_{n,\alpha,\alpha}(f)(x) = \sum\limits_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} \cdot f(\frac{\alpha}{n+\alpha}),\ \forall\ x \in [0,1]$ and Y is closed set (follows using the Jensen's inequalty and passing to limit) and invariant to $P_{n,\alpha,\alpha}$. Since $P_{n,\alpha,\alpha}(e_0) = e_0$ and $P_{n,k}(x) \geqslant 0$, $\forall\ x \in [0,1]$ we infer that:

$$P_{n,k}(x) \in [0,1],\ \forall\ x \in \left[\frac{\alpha}{n+\alpha}, 1\right],\ \forall\ n \in \mathbb{N}^*,\ k = \overline{0,n}$$

and $\sum\limits_{k=0}^{n} P_{n,k}(x) = 1$. We have that

$$P_{n,\alpha,\alpha}(e_1(x)) = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} \cdot \frac{\alpha}{n+\alpha} =$$

$$= x + \frac{\alpha(1-x)}{n+\alpha} \geqslant x = e_1(x),\ \forall\ x \in [0,1].$$

For $f \in Y$ according to the Jensen's inequality, we have:

$$f(x) \leqslant f\left(P_{n,\alpha,\alpha}(e_1)(x)\right) = f\left(\sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} \frac{\alpha}{n+\alpha}\right) \leqslant$$

$$\leqslant \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} f\left(\frac{\alpha}{n+\alpha}\right) = P_{n,\alpha,\alpha}(f)(x), \quad \forall\, x \in \left[\frac{\alpha}{n+\alpha}, 1\right].$$

So,

$$f(x) \leqslant P_{n,\alpha,\alpha}(f)(x), \ \forall\, x \in \left[\frac{\alpha}{n+\alpha}, 1\right], \ \forall\, n \in \mathbb{N}^*.$$

Since $P_{n,\alpha,\alpha}$ is monotone, we infer that:

$$f \leqslant P_{n,\alpha,\alpha}(f) \leqslant P^2_{n,\alpha,\alpha}(f) \leqslant ... \leqslant P^m_{n,\alpha,\alpha}(f) \leqslant ... \leqslant P^{\infty}_{n,\alpha,\alpha}(f) = f(1).$$

b) Here, $P_{n,0,\beta}(x) = \sum\limits_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} f(\frac{n}{n+\beta})$, $\forall\, x \in [0,1]$ and Y is closed and invariant to $P_{n,0,\beta}$. We have that

$$P_{n,0,\beta}(e_1(x)) = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} \frac{n}{n+\beta} = x - \frac{\beta x}{x+\beta}$$

$$\leqslant x = e_1(x), \ \forall x \in [0,1].$$

For $f \in Y$, according to the Jensen's inequality, we have:

$$f(x) \geqslant f\left(P_{n,0,\beta}(e_1)(x)\right) = f\left(\sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^k \frac{n}{n+\beta}\right) \geqslant$$

$$\geqslant \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^k f\left(\frac{n}{n+\beta}\right) = P_{n,0,\beta}(f)(x), \quad \forall\, x \in \left[0, \frac{n}{n+\beta}\right].$$

So,

$$f(x) \geqslant P_{n,0,\beta}(f)(x), \ \forall\, x \in \left[0, \frac{n}{n+\beta}\right], \ \forall\, n \in \mathbb{N}^*.$$

Since $P_{n,0,\beta}$ is monotone we infer that:

$$f \geqslant P_{n,0,\beta}(f) \geqslant P^2_{n,0,\beta}(f) \geqslant ... \geqslant P^m_{n,0,\beta}(f) \geqslant ... \geqslant P^{\infty}_{n,0,\beta}(f) = f(0).$$

□

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