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STRONG CONVERGENCE THEOREMS OF AVERAGING ITERATIONS OF NONEXPANSIVE NONSELF-MAPPINGS IN BANACH SPACES

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Abstract. Let E be a uniformly convex Banach space whose norm is Gâteaux differentiable and which has a weakly continuous duality mapping; for example, every l^p $(1 < p < \infty)$ space has a weakly continuous duality map with gauge function $\varphi(t) = t^{p-1}$. Let C be a nonempty closed convex subset of E, $T : C \to E$ be a nonexpansive nonself-mapping, and x_0, x, y_0, y be elements of C. In this paper, we study the strong convergence of two sequences generated by

$$x_{n+1} = \frac{1}{n+1}\sum_{j=0}^{n}(\alpha_n x + (1-\alpha_n)(PT)^j x_n) \text{ for } n = 0, 1, 2, ...,$$

$$y_{n+1} = \frac{1}{n+1}\sum_{j=0}^{n} P(\alpha_n y + (1-\alpha_n)(TP)^j y_n) \text{ for } n = 0, 1, 2, ...,$$

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where $\{\alpha_n\}$ is a real sequence such that $0 \leq \alpha_n \leq 1$, and P is a sunny and nonexpansive retraction of E onto C.

Key Words and Phrases: Fixed point, nonexpansive nonself-mapping, strong convergence, sunny and nonexpansive retraction, Banach space.

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