

REMARKS ON THE STABILITY OF MONOMIAL FUNCTIONAL EQUATIONS

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Abstract. The aim of this paper is to prove the generalized stability in the Ulam-Hyers-Bourgin sense of the monomial equations, for functions from 2-divisible groups into complete β -normed spaces. There is used a fixed point method, previously applied by the authors to some particular functional equations. A special case, the stability of type Aoki-Rassias is emphasized in section 3 and finally we give some (counter)examples in order to clarify the role of each of the control conditions.

Key Words and Phrases: Monomial functional equation, fixed point, stability.

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