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ADDENDUM TO THE PAPER: SOME RESULTS ON KIRK'S ASYMPTOTIC CONTRACTIONS

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The author recently became aware that Corollary 2.3 in [2] was already proved by T. Suzuki in [3]. In addition it follows from the proof of Theorem 3 in [3] that for any metric space (X, d) and any asymptotic contraction in the sense of Kirk $f : X \to X$, any Picard iteration sequence is Cauchy. However, other results in [2] remain new, namely the explicit and fully uniform rate of convergence in Proposition 2.6.

Suzuki also approaches the problem of removing the assumption of a bounded iteration sequence by modifying Kirk's definition of an asymptotic contraction, proving that the new definition covers the old, and by then proving the relevant theorems for the new class of mappings. However, the modified definition introduced by Suzuki is not very similar to the definitions in [2] and [1]. In the very interesting paper [4] Suzuki introduces the concept of an *asymptotic contraction of the final type* (ACF) and proves a fixed point theorem for these. Suzuki also shows that the definition of an ACF covers the new definition appearing in [3], and also that the ACFs on a metric space (X, d) are exactly the mappings $f : X \to X$ s.t. for all $x, y \in X$ we have that $\lim_{n\to\infty} d(f^n(x), f^n(y)) = 0$ and that $(f^n(x))$ is Cauchy. The concept of an ACF is thus strictly more general than that of a generalized asymptotic contraction in the sense of [2] and [1], since the ACFs on nonempty, complete, bounded metric spaces (X, d) then are the mappings $f : X \to X$ s.t. there exists $z \in X$ s.t. for all $x \in X$ we have that $(f^n(x))$ converges to z. Whereas Proposition 3.7 in [1] shows that generalized asymptotic contractions (or asymptotic contractions in the sense of Kirk) on nonempty, complete, bounded metric spaces (X, d) are exactly the mappings $f : X \to X$ s.t. there exists $z \in X$ s.t. for all $x \in X$ we have that $(f^n(x))$ converges to z with a rate of convergence which is uniform in x. This then highlights what is still new in [2], i.e. the quantitative analysis and the uniformity results. As a consequence of Suzuki's characterization any rate of convergence for ACFs would in a sense depend essentially on x and not just on e.g. strictly positive upper and lower bounds on $d(x, f(x))$. The corresponding situation for the version of asymptotic contractions introduced in [3] is not clear.

References

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