

**APPROACHING NONLINEAR VOLTERRA NEUTRAL
DELAY INTEGRO-DIFFERENTIAL EQUATIONS WITH THE
PEROV'S FIXED POINT THEOREM**

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Abstract. Using the Perov's fixed point theorem and a Bielecki's type norm on a functional space, here we prove the existence and uniqueness of the solution of a class of nonlinear Volterra neutral delay integro-differential equations. Afterwards, we obtain some Lipschitz properties and the error estimation in the approximation of the solution and of his derivative, by the sequence of successive approximations.

Key Words and Phrases: Vector valued generalized metric, Bielecki's norm, Volterra neutral delay integro-differential equation, Perov's fixed point theorem, sequence of successive approximations.

2000 Mathematics Subject Classification: 47H10, 45J05.

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Received: September 29, 2005; Accepted: April 20, 2007.