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STRONG CONVERGENCE THEOREMS FOR A FAMILY OF RELATIVELY NONEXPANSIVE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we deal with the problem of finding a common fixed point of a family of relatively nonexpansive mappings. We, first of all, discuss the properties of strongly relatively nonexpansive mappings and show a strong convergence theorem for a sequence of relatively nonexpansive mappings under some conditions. Using this result, we obtain a strong convergence theorem for a finite family of relatively nonexpansive mappings. Furthermore, we apply our result to the problem of finding a zero of a maximal monotone operator.

Key Words and Phrases: (strongly) relatively nonexpansive mapping, fixed point, maximal monotone operator, resolvent

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