

POSITIVE SOLUTIONS FOR ϕ -LAPLACIAN DIRICHLET BVPS

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Abstract. In this paper, we present new existence results of nontrivial positive solutions for ϕ -Laplacian Dirichlet boundary value problems on bounded intervals of the real line. The nonlinear terms encompasses the sub-linear and super-linear cases. The Krasnosel'skii's fixed point theorem on cone expansion and compression is used. Applications to p -Laplacian problems and to the case of the sum of p -Laplacian and q -Laplacian ($p \neq q$) operators are given.

Key Words and Phrases: ϕ -Laplacian, BVP, positive solution, cone, fixed point theorem.

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