

## A COMPARISON OF PICARD AND MANN ITERATIONS FOR QUASI-CONTRACTION MAPS

J. O. OLALERU

Mathematics Department

University of Lagos

Lagos, Nigeria

E-mail: olaleru1@yahoo.co.uk

**Abstract.** For a class of quasi-contractive operators defined on an arbitrary Banach space, it has been shown that the Picard iteration technique converges faster than the Mann iteration technique. In this paper we make a comparison of the Picard and Mann iterations with respect to their convergence rate for a more general class of operators called quasi-contractions in metrizable topological vector spaces. It was observed that the Picard iteration converges faster than the Mann iteration for this class of maps. This answers the question posed by Berinde in his paper.

**Key Words and Phrases:** topological vector space, fixed point, quasi-contraction, Picard iteration, Mann iteration.

**2000 Mathematics Subject Classification:** 47H10, 46A16.

### REFERENCES

- [1] N. Adasch, B. Ernst, D. Keim, *Topological Vector Spaces*, Springer-Verlag, 1978.
- [2] V. Berinde, *Approximating fixed points of Lipschitzian pseudocontractions*, in Mathematics and Mathematics Education (Bethlehem), 2000, World Scientific Publishing, New Jersey, 2002, 73-81.
- [3] V. Berinde, *On the convergence of the Ishikawa iteration in the class of quasi-contractive operators*, Acta Math. Univ. Comen., **73**2004, 119-126.
- [4] V. Berinde, *Picard iteration converges faster than Mann iteration for a class of quasi-contractive operators*, Fixed Point Theory and Applications, **1**(2004), 1-9.
- [5] S. K. Chatterjea, *Fixed point theorems*, C. R. Acad. Bulgare Sci. **25**(1972), 727-730.
- [6] C. E. Chidume, *Iterative algorithms for nonexpansive mappings and some of their generalisations*, In Nonlinear Analysis and Applications: to V. Lakshmikantham on his 80th birthday. vol. 1-2, Kluwer Acad. Publ., Dordrecht (2003).

- [7] C. E. Chidume, *Geometric properties of Banach spaces and nonlinear iterations*, Monograph, International Centre for Theoretical Physics, Italy, in print by Springer-Verlag.
- [8] L. B. Ćirić, *A generalisation of Banach's contraction principle*, Proc. Amer. Math. Soc., **45**(1974), 267-273.
- [9] L. B. Ćirić, *Convergence theorems for a sequence of Ishikawa iterations for nonlinear quasi-contractive mappings*, Indian J. Pure Appl. Math., **30**(1999), 425-433.
- [10] G. Hardy, T. Rogers, *A generalization of a fixed point theorem of Reich*, Canad. Math. Bull., **16**(1973), 201-206.
- [11] R. Kannan, *Some results on fixed points II*, Amer. Math. Monthly, **76**(1969), 405-408.
- [12] R. Kannan, *Some results on fixed points III*, Fund. Math., **70**(1971), 169-177.
- [13] G. Kothe, *Topological Vector Spaces I*, Springer-Verlag, 1969.
- [14] J. O. Olaleru, *On the convergence of Mann iteration scheme in locally convex spaces*, Carpathian Journal of Mathematics, **22**(2006), 115-120.
- [15] J. O. Olaleru, *On Kannan maps in locally convex spaces*, Proceeding, International Conference on "New Trends in Mathematical and Computer Sciences with Applications to Real World Problems", Covenant University, Cannanland, (2006), 573-585.
- [16] J. O. Olaleru, *Fixed points of quasi-contraction maps in topological vector spaces*, submitted.
- [17] S. Reich, *Some remarks concerning contraction mappings*, Canad. Math. Bull. **14**(1971), 121-124.
- [18] B. E. Rhoades, *Fixed point iterations using infinite matrices*, Trans. Amer. Math. Soc., **196**(1974), 161-176.
- [19] B. E. Rhoades, *Comments on two fixed points iteration Methods*, J. Math. Anal. Appl., **56**(1976), 741-750.
- [20] B. E. Rhoades and S. M. Soltuz, *The equivalence between the Mann and Ishikawa iterations dealing with generalised contractions*, International Journal of Mathematics and Mathematical Sciences, Volume 2006, Article ID 54653, Pages 1-5, 2006.
- [21] V. M. Sehgal, *On fixed and periodic points for a class of mappings*, J. London Math. Soc. **5**(1972), 571-576.
- [22] T. Zamfirescu, *Fixed point theorems in metric spaces*, Arch. Math., Basel, **23**(1972), 292-298.

*Received: October 10, 2006; Accepted: December 8, 2007.*