

## ON SOME GENERALIZATIONS OF THE LANDESMAN-LAZER THEOREM

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**Abstract.** We apply the coincidence degree theory for compact multivalued perturbations of Fredholm operators to obtain necessary and sufficient conditions for the existence of solutions for an equation containing a linear Fredholm operator with an one-dimensional kernel and a discontinuous nonlinearity. Further we consider the extension to the case when the kernel is multi-dimensional and the Fredholm operator is not necessarily self-adjoint. Some examples are given.

**Key Words and Phrases:** Landesman–Laser equation, Fredholm operator, resonance, degeneracy, discontinuous nonlinearity, multivalued map, coincidence point, coincidence degree, topological degree.

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