Fixed Point Theory, Volume 8, No. 1, 2007, 17-27
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

SOME RESULTS ON KIRK'S ASYMPTOTIC CONTRACTIONS

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Abstract. In [W.A. Kirk, Fixed points of asymptotic contractions, J. Math. Anal. Appl. 277 (2003) 645-650], W.A. Kirk proved a fixed point theorem for asymptotic contractions $f : X \to X$ on complete metric spaces (X, d). In the theorem it is assumed that some Picard iteration sequence $(f^n(x_0))$ is bounded. Here we prove that for an asymptotic contraction $f : X \to X$ on a metric space (X, d) any Picard iteration sequence is bounded, thus making the above mentioned assumption of the theorem superfluous. We also provide, given an asymptotic contraction $f : X \to X$ on a complete metric space (X, d), an explicit rate of convergence for a Picard iteration sequence $(f^n(x_0))$ which does not depend on a bound on the iteration sequence, but which instead depends on (strictly positive) upper and lower bounds on $d(x_0, f(x_0))$. This is thus in a sense an improvement on the results in [E.M. Briseid, A rate of convergence for asymptotic contractions, J. Math. Anal. Appl., 330 (2007) 364-376], where the rate of convergence depends on a bound on the iteration sequence. In both cases the rate of convergence also depends on some moduli for the mapping appearing as parameters, but is again in both cases otherwise fully uniform. We can also easily adapt to the situation where the space is not complete.
Key Words and Phrases: metric fixed point theory, asymptotic contractions, proof mining.
2000 Mathematics Subject Classification: 47H10, 47H09, 03F10.

Acknowledgements

I would like to express my gratitude to Ulrich Kohlenbach for stimulating discussions concerning the subject matter.

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Received: August 7, 2006; Accepted: October 10, 2006.