

STRONG CONVERGENCE THEOREMS ON THE MODIFIED ITERATIVE ALGORITHM FOR A FAMILY OF FINITE NONEXPANSIVE NONSELF MAPPINGS

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Abstract. Let K be a nonempty closed convex subset of a real Banach space E which has a uniformly Gâteaux differentiable norm. Assume that K is a sunny nonexpansive retract of E with Q as the sunny nonexpansive retraction. Let $T_i : K \rightarrow E, i = 1, 2, \dots, N$ be a family of nonexpansive mappings which are weakly inward with $F = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $f : K \rightarrow K$ be a fixed contractive mapping. For given $x_0 \in K$, let $\{x_n\}$ be generated by the algorithm

$$x_{n+1} = \alpha_n f(x_n) + \beta_n x_n + \gamma_n Q T_{n+1} x_n, \quad n \geq 0.$$

Some sufficient and necessary conditions are proved for a common fixed point of a family of nonexpansive mappings provided $T_i, i = 1, 2, \dots, N$ satisfy some mild conditions.

Key Words and Phrases: Nonexpansive mapping, strong convergence, common fixed point, uniformly Gâteaux differentiable norm

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