

POSITIVE SOLUTIONS OF SINGULAR SUBLINEAR SECOND-ORDER THREE-POINT BOUNDARY VALUE PROBLEMS

RUYUN MA* and DONAL O'REGAN**

*Department of Mathematics
Northwest Normal University
Lanzhou 730070, P R China

**Department of Mathematics
National University of Ireland
Galway, Ireland
E-mail: donal.oregan@nuigalway.ie

Abstract. We give some necessary and sufficient conditions for the existence of C or C^1 positive solutions of the singular boundary value problem

$$\begin{aligned}x''(t) + p(t)x^\lambda(t) &= 0, & t \in (0, 1) \\ x(0) = 0, \quad x(1) &= \alpha x(\eta)\end{aligned}$$

where $\eta \in (0, 1)$, $\alpha \in (0, 1]$ and $\lambda \in (0, 1)$ are given, $p : (0, 1) \rightarrow [0, \infty)$ can be singular at both ends $t = 0$ and $t = 1$. The main tool is the method of lower and upper solutions for singular three-point boundary value problems.

Key Words and Phrases: Singular boundary value problem, Existence, Schauder fixed point theorem, Green's function, Lower and upper solution.

2000 Mathematics Subject Classification: 34B10, 34B18.

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Supported by the NSFC (No. 10271095), GG-110-10736-1003, Spring-Sun Program (No. Z2004-1-62033), the Foundation of Excellent Young Teacher of the Chinese Education Ministry.

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Received 19.06.2005, Revised 10.12.2005