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FIXED POINTS AND COMMON FIXED POINTS FOR SOME MULTIVALUED OPERATORS

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Abstract. We present an answer to the following problem. Let (X, d) be a metric space and $T_1, T_2 : X \to P(X)$ two multivalued operators. Determine metric conditions on the pair of multivalued operators T_1 and T_2, which imply that for each $x \in X$ there exists a sequence of successive approximations for the pair (T_1, T_2) or for the pair (T_2, T_1), starting from x, which converges to a common fixed point or to a common strict fixed point of T_1 and T_2 and for each $x \in X$ there exists a sequence of successive approximations of T_i, starting from x, which converges to a fixed point or to a strict fixed point of T_i, for each $i \in \{1, 2\}$. We also prove that the common fixed points set of two multifunctions $T_1, T_2 : \mathbb{R} \to P_{cp,\ cv}(\mathbb{R})$, which satisfy a contraction type condition, is a compact and convex set.

Key Words and Phrases: multivalued operator, fixed point, strict fixed point, common fixed point.
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1. Introduction

Let X be a nonempty set.

We denote by $P(X)$ the set of all nonempty subsets of X, i. e. $P(X) := \{\ Y \mid \emptyset \neq Y \subseteq X\ \}$. Let $f : X \to X$ be a singlevalued operator and $T_1, T_2 : X \to P(X)$ two multivalued operators. We denote by F_f the fixed points set of f, i. e. $F_f := \{\ x \in X \mid f(x) = x\ \}$, by F_{T_1} the fixed points set of T_1, i. e. $F_{T_1} := \{\ x \in X \mid x \in T_1(x)\ \}$, by $(SF)_{T_1}$ the strict fixed points set of T_1, i. e. $(SF)_{T_1} := \{\ x \in X \mid T_1(x) = \{x\}\ \}$ and by $(CF)_{T_1,T_2}$ the common fixed points set, i.e. $(CF)_{T_1,T_2} := \{\ x \in X \mid x \in T_1(x) \cap T_2(x)\ \}$.

A sequence $(x_n)_{n\in\mathbb{N}}$ is called *sequence of successive approximations of* T_1 if $x_0 \in X$ and $x_{n+1} \in T_1(x_n)$, for each $n \in \mathbb{N}$.

A sequence $(x_n)_{n\in\mathbb{N}}$ is called *sequence of successive approximations for the pair* (T_1, T_2) if $x_0 \in X$, $x_{2n+1} \in T_1(x_{2n})$ and $x_{2n+2} \in T_2(x_{2n+1})$, for each $n \in \mathbb{N}$.

Let (X, d) be a metric space.

We denote by $P_b(X)$ the set of all nonempty and bounded subsets of X, i. e. $P_b(X) := \{\ Y \mid Y \in P(X),\ Y$ is a bounded set $\}$ and by $P_{cp,\ cv}(X)$ the set of all nonempty, compact and convex subsets of X, i. e. $P_{cp,\ cv}(X) := \{\ Y \mid Y \in P(X),\ Y$ is a compact and convex set $\}$.

We also recall the functional $D : P(X)\times P(X) \to \mathbb{R}_+$, defined by $D(A, B) = \inf\ \{\ d(a, b) \mid a \in A,\ b \in B\ \}$, for each $A, B \in P(X)$, and the generalized functionals $\delta : P(X)\times P(X) \to \mathbb{R}_+\cup\{+\infty\}$, defined by $\delta(A, B) = \sup\ \{\ d(a, b) \mid a \in A,\ b \in B\ \}$, for each $A, B \in P(X)$, and $H : P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$, defined by $H(A, B) = \max\ \{\ \sup_{a\in A} D(a, B),\ \sup_{b\in B} D(b, A)\ \}$, for each $A, B \in P(X)$.

2. Some pairs of multivalued operators

There are many strict fixed point and common strict fixed point theorems for multivalued operators which satisfy metric conditions in which functional δ appears (see, for example, Reich [11], Ćirić [3], [5], Rus [12], Avram [2], Fisher [7], Khan-Khan-Kubiaczyk [8], Dien [6], Kubiaczyk [10], Khan-Kubiaczyk [9]).

In this section is studied the following problem.

Problem 2.1. *Let (X, d) be a metric space and $T_1, T_2 : X \to P(X)$ two multivalued operators. Determine metric conditions on the pair of multivalued operators T_1 and T_2, which imply that for each $x \in X$ there exists a sequence of successive approximations for the pair (T_1, T_2) or for the pair (T_2, T_1), starting from x, which converges to a common fixed point or to a common strict fixed point of T_1 and T_2 and for each $x \in X$ there exists a sequence of successive approximations of T_i, starting from x, which converges to a fixed point or to a strict fixed point of T_i, for each $i \in \{1, 2\}$.*

For singlevalued operators results of this type are given by Rus [13] and Dien [6] and for multivalued operators results which answer to Problem 2.1 are presented by Sîntămărian [16], [17].

There is also an interesting result of this kind given by Dien [6], for two multivalued operators which satisfy a metric condition in which functional δ appears.

The following result gives another answer to Problem 2.1 for two multivalued operators which satisfy a metric condition in which functional δ appears.

Theorem 2.1. *Let (X,d) be a complete metric space and $T_1, T_2 : X \to P_b(X)$ two multivalued operators for which there exists $a \in [0,1/2[$ such that*

$$\delta(T_1(x), T_2(y)) \leq a\ [\delta(x, T_1(x)) + \delta(y, T_2(y))],$$

for each $x, y \in X$.

Then $F_{T_1} = F_{T_2} = (SF)_{T_1} = (SF)_{T_2} = \{x^*\}$ and, for each $i, j \in \{1,2\}$, with $i \neq j$, any sequence $(x_n)_{n\in\mathbb{N}}$ of successive approximations for the pair (T_i, T_j) converges to x^* and

$$d(x_n, x^*) \leq \frac{1-a}{1-2a}\left(\frac{a}{1-a}\right)^n \delta(x_0, T_i(x_0)),$$

for every $n \in \mathbb{N}$.

Also, for each $i \in \{1,2\}$, any sequence $(y_n)_{n\in\mathbb{N}}$ of successive approximations of T_i converges to x^* and

$$d(y_n, x^*) \leq \frac{1-a}{1-2a}\left(\frac{a}{1-a}\right)^n \delta(y_0, T_i(y_0)),$$

for every $n \in \mathbb{N}$.

Proof. The fact that T_1 and T_2 have a unique common fixed point, which is a strict fixed point both of T_1 and of T_2, it is a known result. In order to prove some other parts of the conclusion we shall take again the proof.

Let $i, j \in \{1,2\}$, $i \neq j$. Let $x_0 \in X$, $x_{2n-1} \in T_i(x_{2n-2})$ and $x_{2n} \in T_j(x_{2n-1})$, for each $n \in \mathbb{N}^*$.

We have

$$\delta(T_i(x_0), T_j(x_1)) \leq a\ [\delta(x_0, T_i(x_0)) + \delta(x_1, T_j(x_1))] \leq$$

$$\leq a\ [\delta(x_0, T_i(x_0)) + \delta(T_i(x_0), T_j(x_1))]$$

and so

$$d(x_1, x_2) \leq \delta(T_i(x_0), T_j(x_1)) \leq a/(1-a)\ \delta(x_0, T_i(x_0)).$$

For each $n \in \mathbb{N}^*$ we have

$$\delta(T_j(x_{2n-1}), T_i(x_{2n})) \leq a\ [\delta(x_{2n-1}, T_j(x_{2n-1})) + \delta(x_{2n}, T_i(x_{2n}))] \leq$$

$$\leq a\ [\delta(T_i(x_{2n-2}), T_j(x_{2n-1})) + \delta(T_j(x_{2n-1}), T_i(x_{2n}))]$$

and from here we get that

$$d(x_{2n}, x_{2n+1}) \leq \delta(T_j(x_{2n-1}), T_i(x_{2n})) \leq a/(1-a)\ \delta(T_i(x_{2n-2}), T_j(x_{2n-1})).$$

Also, for each $n \in \mathbb{N}^*$ we have

$$\delta(T_i(x_{2n}), T_j(x_{2n+1})) \leq a\ [\delta(x_{2n}, T_i(x_{2n})) + \delta(x_{2n+1}, T_j(x_{2n+1}))] \leq$$

$$\leq a\ [\delta(T_j(x_{2n-1}), T_i(x_{2n})) + \delta(T_i(x_{2n}), T_j(x_{2n+1}))]$$

and so

$$d(x_{2n+1}, x_{2n+2}) \leq \delta(T_i(x_{2n}), T_j(x_{2n+1})) \leq a/(1-a)\ \delta(T_j(x_{2n-1}), T_i(x_{2n})).$$

Now, we are able to write that

$$d(x_n, x_{n+1}) \leq [a/(1-a)]^n\ \delta(x_0, T_i(x_0)),$$

for each $n \in \mathbb{N}$.

Let $p \in \mathbb{N}^*$. Using the triangle inequality we obtain

$$d(x_n, x_{n+p}) \leq (1-a)/(1-2a)[a/(1-a)]^n\ \delta(x_0, T_i(x_0)),$$

for each $n \in \mathbb{N}$. It follows that $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence and so a convergent sequence, because (X, d) is a complete metric space. Let $x^* = \lim_{n\to\infty} x_n$.

Letting p to tend to infinity in the above inequality we get that

$$d(x_n, x^*) \leq (1-a)/(1-2a)[a/(1-a)]^n\ \delta(x_0, T_i(x_0)),$$

for every $n \in \mathbb{N}$.

We have

$$\delta(x^*, T_i(x^*)) \leq d(x^*, x_{2n+2}) + \delta(x_{2n+2}, T_i(x^*)) \leq$$

$$\leq d(x^*, x_{2n+2}) + \delta(T_j(x_{2n+1}), T_i(x^*)) \leq$$

$$\leq d(x^*, x_{2n+2}) + a\ [\delta(x_{2n+1}, T_j(x_{2n+1})) + \delta(x^*, T_i(x^*))] \leq$$

$$\leq d(x^*, x_{2n+2}) + a\ [\delta(T_i(x_{2n}), T_j(x_{2n+1})) + \delta(x^*, T_i(x^*))] \leq$$

$$\leq d(x^*, x_{2n+2}) + a\ \{[a/(1-a)]^{2n+1}\ \delta(x_0, T_i(x_0)) + \delta(x^*, T_i(x^*))\},$$

for all $n \in \mathbb{N}$.

From this we get that

$$\delta(x^*, T_i(x^*)) \leq 1/(1-a)\ \{d(x^*, x_{2n+2}) + a[a/(1-a)]^{2n+1}\ \delta(x_0, T_i(x_0))\},$$

for each $n \in \mathbb{N}$.

Letting n to tend to infinity it follows that $\delta(x^*, T_i(x^*)) = 0$, so $T_i(x^*) = \{x^*\}$. It is easy to verify that $(CF)_{T_1,T_2} = (SF)_{T_1} = (SF)_{T_2} = \{x^*\}$.

In order to prove that $F_{T_i} = \{x^*\}$, let $x \in F_{T_i}$. Then we have

$$\delta(x, T_i(x)) \leq \delta(T_i(x), T_i(x)) \leq \delta(T_i(x), T_j(x^*)) + \delta(T_j(x^*), T_i(x)) \leq$$

$$\leq a\,[\delta(x, T_i(x)) + \delta(x^*, T_j(x^*))] + a\,[\delta(x^*, T_j(x^*)) + \delta(x, T_i(x))] = 2a\,\delta(x, T_i(x)).$$

From this we get that $\delta(x, T_i(x)) = 0$, so $T_i(x) = \{x\}$, i. e. $x \in (SF)_{T_i}$.

Let $y_0 \in X$ and $y_{n+1} \in T_i(y_n)$, for each $n \in \mathbb{N}$. We have

$$\delta(T_i(y_0), T_i(y_1)) \leq \delta(T_i(y_0), T_j(x^*)) + \delta(T_j(x^*), T_i(y_1)) \leq$$

$$\leq a\ [\delta(y_0, T_i(y_0)) + \delta(x^*, T_j(x^*))] + a\ [\delta(x^*, T_j(x^*)) + \delta(y_1, T_i(y_1))] =$$

$$= a\ [\delta(y_0, T_i(y_0)) + \delta(y_1, T_i(y_1))] \leq a\ [\delta(y_0, T_i(y_0)) + \delta(T_i(y_0), T_i(y_1))],$$

which implies

$$\delta(T_i(y_0), T_i(y_1)) \leq a/(1-a)\ \delta(y_0, T_i(y_0)).$$

Also, for each $n \in \mathbb{N}^*$ we have

$$\delta(T_i(y_n), T_i(y_{n+1})) \leq \delta(T_i(y_n), T_j(x^*)) + \delta(T_j(x^*), T_i(y_{n+1})) \leq$$

$$\leq a\ [\delta(y_n, T_i(y_n)) + \delta(x^*, T_j(x^*))] + a\ [\delta(x^*, T_j(x^*)) + \delta(y_{n+1}, T_i(y_{n+1}))] =$$

$$= a\ [\delta(y_n, T_i(y_n)) + \delta(y_{n+1}, T_i(y_{n+1}))] \leq$$

$$\leq a\ [\delta(T_i(y_{n-1}), T_i(y_n)) + \delta(T_i(y_n), T_i(y_{n+1}))]$$

and hence

$$\delta(T_i(y_n), T_i(y_{n+1})) \leq a/(1-a)\ \delta(T_i(y_{n-1}), T_i(y_n)).$$

It follows that

$$\delta(T_i(y_{n-1}), T_i(y_n)) \leq [a/(1-a)]^n\ \delta(y_0, T_i(y_0)),$$

for each $n \in \mathbb{N}^*$ and hence

$$d(y_n, y_{n+1}) \leq [a/(1-a)]^n\ \delta(y_0, T_i(y_0)),$$

for each $n \in \mathbb{N}$.

As way stated above, we obtain that

$$d(y_n, y_{n+p}) \leq (1-a)/(1-2a)[a/(1-a)]^n \ \delta(y_0, T_i(y_0)),$$

for each $n \in \mathbb{N}$ and for every $p \in \mathbb{N}^*$.

From this we get that $(y_n)_{n\in\mathbb{N}}$ is a Cauchy sequence and so a convergent sequence, because (X, d) is a complete metric space. Let $y^* = \lim_{n\to\infty} y_n$.

Letting p to tend to infinity in the above inequality we obtain that

$$d(y_n, y^*) \leq (1-a)/(1-2a)[a/(1-a)]^n \ \delta(y_0, T_i(y_0)),$$

for each $n \in \mathbb{N}$.

We have

$$\delta(y^*, T_i(y^*)) \leq d(y^*, y_{n+1}) + \delta(y_{n+1}, T_i(y^*)) \leq d(y^*, y_{n+1}) + \delta(T_i(y_n), T_i(y^*)) \leq$$
$$\leq d(y^*, y_{n+1}) + \delta(T_i(y_n), T_j(x^*)) + \delta(T_j(x^*), T_i(y^*)) \leq$$
$$\leq d(y^*, y_{n+1}) + a\ [\delta(y_n, T_i(y_n)) + \delta(x^*, T_j(x^*))] + a\ [\delta(x^*, T_j(x^*)) + \delta(y^*, T_i(y^*))] =$$
$$= d(y^*, y_{n+1}) + a\ [\delta(y_n, T_i(y_n)) + \delta(y^*, T_i(y^*))] \leq$$
$$\leq d(y^*, y_{n+1}) + a\ [\delta(T_i(y_{n-1}), T_i(y_n)) + \delta(y^*, T_i(y^*))] \leq$$
$$\leq d(y^*, y_{n+1}) + a\ \{[a/(1-a)]^n\ \delta(y_0, T_i(y_0)) + \delta(y^*, T_i(y^*))\},$$

for all $n \in \mathbb{N}^*$.

From this we obtain

$$\delta(y^*, T_i(y^*)) \leq 1/(1-a)\ \{d(y^*, y_{n+1}) + a[a/(1-a)]^n\ \delta(y_0, T_i(y_0))\},$$

for each $n \in \mathbb{N}^*$.

Letting n to tend to infinity it follows that $\delta(y^*, T_i(y^*)) = 0$, so $T_i(y^*) = \{y^*\}$. It means that $y^* \in (SF)_{T_i} = \{x^*\}$. ■

Corollary 2.1. *Let (X, d) be a complete metric space and $T_1, T_2 : X \to P_b(X)$ two multivalued operators for which there exists $a \in [0, 1/2[$ such that*

$$\delta(T_1(x), T_2(y)) \leq a\ [\delta(x, T_1(x)) + \delta(y, T_2(y))],$$

for each $x, y \in X$.

Then $F_{T_1} = F_{T_2} = (SF)_{T_1} = (SF)_{T_2} = \{x^*\}$ and

$$d(x_0, x^*) \leq (1-a)/(1-2a)\ \min\ \{\delta(x_0, T_1(x_0)), \delta(x_0, T_2(x_0))\},$$

for each $x_0 \in X$.

Proof. We take $n = 0$ in Theorem 2.1. ■

Example 2.1. *Let* $T_1 : \mathbb{R} \to P_b(\mathbb{R})$ *defined by*

$$T_1(x) = \begin{cases} \left[-\frac{x}{16}, -\frac{x}{8}\right], & \textit{if} \quad x < 0, \\ \{0\}, & \textit{if} \quad x = 0, \\ \left[-\frac{x}{8}, -\frac{x}{16}\right], & \textit{if} \quad x > 0, \end{cases}$$

and let $T_2 : \mathbb{R} \to P_b(\mathbb{R})$ *defined by*

$$T_2(x) = \begin{cases} \left[-\frac{x}{32}, -\frac{x}{16}\right], & \textit{if} \quad x < 0, \\ \{0\}, & \textit{if} \quad x = 0, \\ \left[-\frac{x}{16}, -\frac{x}{32}\right], & \textit{if} \quad x > 0. \end{cases}$$

In order to verify that the inequality

$$\delta(T_1(x), T_2(y)) \le a\ [\delta(x, T_1(x)) + \delta(y, T_2(y))]$$

holds for each $x, y \in \mathbb{R}$*, with* $a = \frac{1}{5} \in \left[0, \frac{1}{2}\right[$*, we consider the following nine cases:*

$$\begin{array}{lll} 1^\circ\ x < 0, y < 0; & 2^\circ\ x < 0, y = 0; & 3^\circ\ x < 0, y > 0; \\ 4^\circ\ x = 0, y < 0; & 5^\circ\ x = 0, y = 0; & 6^\circ\ x = 0, y > 0; \\ 7^\circ\ x > 0, y < 0; & 8^\circ\ x > 0, y = 0; & 9^\circ\ x > 0, y > 0. \end{array}$$

In case 1° *we take the subcases*: *a*) $x \le y < 0$ *and b*) $y < x < 0$.

For the subcase b) *we have*

$$\delta(T_1(x), T_2(y)) \le -\frac{y}{8} + \frac{x}{32} \le \frac{1}{5}\left(-\frac{x}{8} - x - \frac{y}{16} - y\right) =$$

$$= \frac{1}{5}\left[\delta(x, T_1(x)) + \delta(y, T_2(y))\right],$$

for each $x, y \in \mathbb{R}$*, with* $y < x < 0$.

In case 9° *we take the subcases*: *a*) $0 < x < y$ *and b*) $0 < y \le x$.

For the subcase a) *we have*

$$\delta(T_1(x), T_2(y)) \le -\frac{x}{32} + \frac{y}{8} \le \frac{1}{5}\left(x + \frac{x}{8} + y + \frac{y}{16}\right) =$$

$$= \frac{1}{5}\left[\delta(x, T_1(x)) + \delta(y, T_2(y))\right],$$

for each $x, y \in \mathbb{R}$*, with* $0 < x < y$.

In rest it is not difficult to see that the inequality is satisfied.

It is clear that $F_{T_1} = F_{T_2} = (SF)_{T_1} = (SF)_{T_2} = \{0\}$.

3. Some properties of the common fixed points set of two multifunctions

In [1] and [15] are studied some properties of the fixed points set of a multifunction, which are inherited from the values of the multifunction.

Regarding the properties of the common fixed points set of two multifunctions, which are inherited from the values of the multifunctions, we give the following result.

Theorem 3.1. *Let* $T_1, T_2 : \mathbb{R} \to P_{cp,\ cv}(\mathbb{R})$ *be two multifunctions. We suppose that there exists* $a \in [0, 1[$ *such that*

$$H(T_1(x), T_2(y)) \leq a \ \max \{ \ |x-y|, D(x, T_1(x)), D(y, T_2(y)), \\ 1/2 \ [D(x, T_2(y)) + D(y, T_1(x))] \ \},$$

for each $x, y \in \mathbb{R}$.

Then $F_{T_1} = F_{T_2} \in P_{cp,\ cv}(\mathbb{R})$.

Proof. For every $x \in \mathbb{R}$ we have $T_1(x), T_2(x) \in P_{cp,\ cv}(\mathbb{R})$. Hence, there exist $m_1, M_1, m_2, M_2 : \mathbb{R} \to \mathbb{R}$ so that $T_1(x) = [m_1(x), M_1(x)]$ and $T_2(x) = [m_2(x), M_2(x)]$, for each $x \in \mathbb{R}$.

It follows that

$$H(T_1(x), T_2(y)) = \max \{|m_1(x) - m_2(y)|, |M_1(x) - M_2(y)| \ \} \leq$$

$$\leq a \ \max \{ \ |x-y|, D(x, T_1(x)), D(y, T_2(y)), 1/2 \ [D(x, T_2(y)) + D(y, T_1(x))] \ \},$$

for every $x, y \in \mathbb{R}$.

So

$$|m_1(x) - m_2(y)| \leq a \ \max \{ \ |x-y|, |x - m_1(x)|, |y - m_2(y)|, \\ 1/2 \ [|x - m_2(y)| + |y - m_1(x)|] \ \},$$

and

$$|M_1(x) - M_2(y)| \leq a \ \max \{ \ |x-y|, |x - M_1(x)|, |y - M_2(y)|, \\ 1/2 \ [|x - M_2(y)| + |y - M_1(x)|] \ \},$$

for each $x, y \in \mathbb{R}$.

From these, taking into account a result given by Ćirić (Theorem 1 in [4], Theorem 4.5 in [5]), we have that there exists $x_m \in \mathbb{R}$ such that $F_{m_1} = F_{m_2} = \{x_m\}$ and there exists $x_M \in \mathbb{R}$ such that $F_{M_1} = F_{M_2} = \{x_M\}$.

It is not difficult to show that $x_m \leq x_M$.

Also, it is easy to verify that if $x < x_m$, then $x \notin F_{T_1} \cup F_{T_2}$ and if $x > x_M$, then $x \notin F_{T_1} \cup F_{T_2}$.

In case $x_m < x_M$ and $x \in]x_m, x_M[$, then we have $x \in]m_1(x), M_1(x)[\cap]m_2(x), M_2(x)[$.

Therefore, we are able to write that $F_{T_1} = F_{T_2} = [x_m, x_M]$. ■

References

[1] M.-C. Alicu, O. Mark, *Some properties of the fixed points set for multifunctions*, Studia Univ. Babeş-Bolyai, Mathematica, **25**(1980), 77-79.

[2] M. Avram, *Points fixes communs pour les applications multivoques dans les espaces métriques*, Mathematica, **17**(1975), 153-156.

[3] Lj. B. Ćirić, *A generalization of Banach's contraction principle*, Proc. Amer. Math. Soc., **45**(1974), 267-273.

[4] Lj. B. Ćirić, *On a family of contractive maps and fixed points*, Publ. Inst. Math., **17**(1974), 45-51.

[5] Lj. B. Ćirić, *Some Recent Results in Metrical Fixed Point Theory*, Belgrade, 2003.

[6] N. H. Dien, *Some common fixed point theorems in metric spaces*, Seminar on Fixed Point Theory, 1985, 15-36.

[7] B. Fisher, *Set-valued mappings on metric spaces*, Fund. Math. **112**(1981), 141-145.

[8] M. S. Khan, M. D. Khan, I. Kubiaczyk, *Some common fixed point theorems for multi-valued mappings*, Demonstratio Math., **17**(1984), 997-1002.

[9] M. S. Khan, I. Kubiaczyk, *Fixed point theorems for point to set maps*, Math. Japonica, **33**(1988), 409-415.

[10] I. Kubiaczyk, *Fixed points for contractive correspondences*, Demonstratio Math., **20**(1987), 495-500.

[11] S. Reich, *Fixed points of contractive functions*, Boll. U. M. I., **5**(1972), 26-42.

[12] I. A. Rus, *Fixed point theorems for multi-valued mappings in complete metric spaces*, Math. Japonica, **20**(1975), 21-24.

[13] I. A. Rus, *Approximation of common fixed point in a generalized metric space*, L'Anal. Numér. Théor. L'Approx., **8**(1979), 83-87.

[14] I. A. Rus, *Generalized Contractions and Applications*, Cluj University Press, Cluj-Napoca, 2001.

[15] H. Schirmer, *Properties of the fixed point set of contractive multi-functions*, Canad. Math. Bull., **13**(1970), 169-173.

[16] A. Sîntămărian, *Weakly Picard pairs of multivalued operators*, Mathematica, **45**(2003), 195-204.

[17] A. Sîntămărian, *Weakly Picard pairs of some multivalued operators*, Math. Commun., **8**(2003), 49-53.